Chapter 1

An Optimal Strategy for Maintaining Excess Capacity

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Report compiled by Daya Gaur

1.1 Abstract

Boeing is a manufacturing industry with very low production volumes of very large units. As such, they experience huge fluctuations in demand. A standard inventory model dictates massive changes in production capacity as demand varies. However all such models assume a continuous production stream. In this report we investigate the following question whether such a model is sensible in a problem of such large scale granularity. We describe a combination of stochastic, financial and simulation models to model the production of airplanes. A preliminary simulation of the model is also presented.

1.2 Introduction

After the merger with McDonnell Douglas and the acquisition of the defense and space units of Rockwell, Boeing is the largest airline manufacturer in the world, holding 60% of the market. Boeing traditionally experiences wide fluctuation in the plane demand. However, they have good models for long-term projection of future demand. This raises the following question: given projections about the future demand, what should be the schedule for building airplanes so as to maximize profit? The primary cost associated with any scheduling system is the cost of the inventory. Currently, Boeing follows the *Just In Time* production model (zero inventory), meaning that planes are only constructed as ordered.

The objective of this study is to make a case that by maintaining the inventory of partially built planes, both Boeing and the customer can profit: Boeing by minimizing the delivery schedules (which in turn translates into increased revenue) and the customer by not having to wait for up to two years (to start a new route, for example). By having an inventory Boeing would also not face

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the problem of having to continuously hire and fire employees. This is a particularly aggravating problem for Boeing since they employ highly skilled labour force which itself is a scarce commodity.

Although Boeing is part of a manufacturing industry, its volume of production is insignificant compared to that in other manufacturing industries such as the automobile industry. Given the high level of granularity in Boeing's production, the standard models of job shop, flow lines, transfer lines, machining systems etc. Reference [3] cannot be applied directly to model this production of airplanes. The models cited above assume a high volume of production and a continuous demand for the product but neither of these assumptions is valid in Boeing's case.

In the airplane market, typically the time required to produce one plane is 24 months. Customers are willing to pay more for the planes delivered earlier as this gives them financial flexibility in decision making. Hence, were Boeing to keep an inventory of partially built planes they could sell these planes at a higher price and generate more revenue. For the customer the extra money is a strategic investment to have an edge over the competitors (for instance these competitors could not introduce routes in a shorter time frame). In financial terms, this extra worth can be viewed as an *option* to buy a plane by a specific date.

Since the demand for planes is unpredictable, stored planes permit Boeing to capitalize on large upswings in demand. In terms of practicality, this is an easy scheme to implement since the inexpensive, time-consuming steps of building planes are typically done before the expensive, faster steps. For example the hull construction, piping and wiring, are all done before the engines, cockpit and interior decoration are added. We will be analyzing a production system wherein Boeing stores partially built planes at different levels of completion.

The following questions were addressed in some detail at the Second PIMS Industrial Problem Solving Workshop held in Calgary from June 1-5, 1998.

1. What is a reasonable way to model the projected future demand of a plane model?
2. What kind of production model would be best suited to modeling an airplane assembly line? Is it possible to simulate such a system?
3. How should the planes be priced based on the delivery date?

Section 1.3 shows that the future demand of the planes could be modeled as a Wiener process. This conclusion was reached by doing a time series analysis of the projected demand data. Section 1.4 describes the queuing model which is used to maintain the inventory and simulate the airplane production process. It also comments on the inconclusiveness of the preliminary simulation results. Section 1.5 describes how the pricing of the planes is to be done based on the delivery date. The idea is to sell the customer an option to buy a plane instead of selling a plane. Though in reality the option would be a *real option*, during our simulation, for the sake of simplicity we modeled it as *European option*.

Note that the data within this report are hypothetical, or from public sources, and no statement made herein reflects an official policy of the Boeing Corporation.

1.3 Modeling the future demand

Traditionally, it is believed that plane orders follow a regular business cycle. However, in this section we show that in fact the number of airplane orders, denoted Y_t, follows a geometric Brownian motion:

$$Y_t = e^{X_t} \qquad (1)$$

where X_t is a simple Brownian motion.

A series of data obtained from 1955 to 1997 was used in our study. Taking logarithm on both sides of equation 1 we obtain $log Y_t = X_t$. We will show that the logarithm of the data follows a simple Brownian motion. Before delving into the details, we explain why geometric Brownian motion is expected instead of simple Brownian motion. The usual assumptions when one analyzes data are:

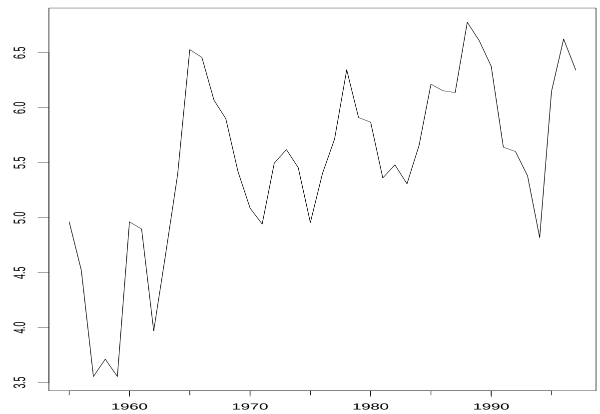

Figure 1.1: Time Series Plot of log of Demand

- Linearity of the model.
- Normality of the observations.
- The constancy of variance.
- Independence of observations.

The assumption about the normality of the data is violated by our data set. If any of these assumptions are not met by the data, then Tukey [6] suggests two alternatives: either a new analysis must be devised to meet the assumptions, or the data must be transformed to meet the usual assumptions. As it is usually easier to transform the data than to develop a new method of analysis, we transform the data of plane orders by a logarithm function, a common transformation used for count data. The time series plot of the transformed data is shown in Figure 1.3.

Brownian motion takes place in continuous time and continuous space. Our attempt to model the data proceeds by approximating it by a discrete process such as a random walk. We define Lag difference as $X_t - X_{t-1}$. The series of Lag difference on the 'log' data, $logY_t - logY_{t-1}$, shows randomness. We define $\epsilon_t = logY_t - logY_{t-1}$, and investigate the pattern formed. We argue that the time series plot of ϵ_t, which is shown in Figure1.3, shows a random pattern.

The sample autocorrelation function (ACF), which is the sample estimate of the autocorrelation function defined as $r(h) = Corr(\epsilon_{t+h}, \epsilon_t)$, shows that ϵ_{t+h} and ϵ_t are uncorrelated for $h = 1, 2, 3, ...$ This can be graphically seen from the ACF plot in Figure 1.3 with the data series $lnDdif = \epsilon_t$. All the sample estimates of autocorrelation, which are corresponding to the vertical bars in the graph, are not significantly different from zero (since all these bars are within the 2-standard-deviation horizontal lines) for Lag (h) greater than zero.

The qq-plot in Figure 1.3, which plots the quantiles of a normal distribution against the quantiles of the data, suggests that the ϵ_t are normally distributed. This normality is further supported by the modified version of qq-plot for χ^2 distribution. The agreement of normality and zero correlation of ϵ_t implies independence. Now we reach the conclusion that X_t or $logY_t$ follows a Random Walk. The estimates of the mean and standard deviation of the ϵ_t are 0 and 0.3, respectively, where the zero mean is suggested by the t-test.

Finally, we point out that this by no means is conclusive evidence that the model of geometric Brownian motion is an optimal model for the series of data. Rather we have only established that

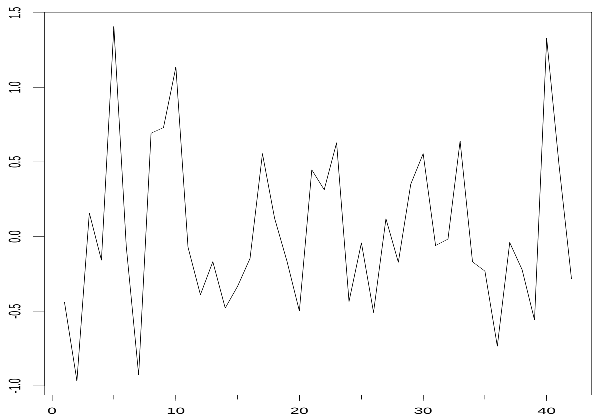

Figure 1.2: Time series plot of ϵ_t

there are no strong violations to the assumption that the input data follows the geometric Brownian motion.

1.4 Queuing Model

We argued in Section 1.2 that the production of planes cannot be modeled using job shop, assembly lines or flow line models of the manufacturing industry due to high level of granularity in the process. The model which most closely resembles our problem is that of multiple cell manufacturing system [3]. The idea is to replace a large job shop by specialized cells producing groups of job types that are identified as being similar in design and production process. Many issues arise in design, planning and operation of a cell model. In our simulation we restrict ourselves only to the operation of a cell model. Two important issues which arise are:

- *Buffer sizes:* Input and output of each cell is modeled using an inventory buffer. The maximum (and minimum) buffer sizes will have a drastic affect on the cost of the production. Our simulation tries to determine the revenue generated based on the Input rate and the Output rate. Input and the Output rates correspond to the buffer sizes.

- *Cell operation:* Each cell has to be designed so that it operates at maximum efficiency and maximum profit. In the simulation model we do not address this question. Standard treatment would be to model each cell as a Jackson queuing network [4].

We propose a combination of stochastic and simulation model to model Boeing's production process using multiple cells. Our model, shown in Figure 1.5, comprises two cells (specialized job shops) which produce planes that are 50% and 100% assembled respectively. Both the cells are producing planes at some specified rate. Planes which are produced by cell 1 are completed in cell 2. For the purpose of our simulation there is a server which keeps track of the incoming requests and services them based on the availability of the planes in cell 2.

The following facts were used in generating the simulation model.

- The demand is stochastic in nature that is geometric Brownian (see Section 1.3).

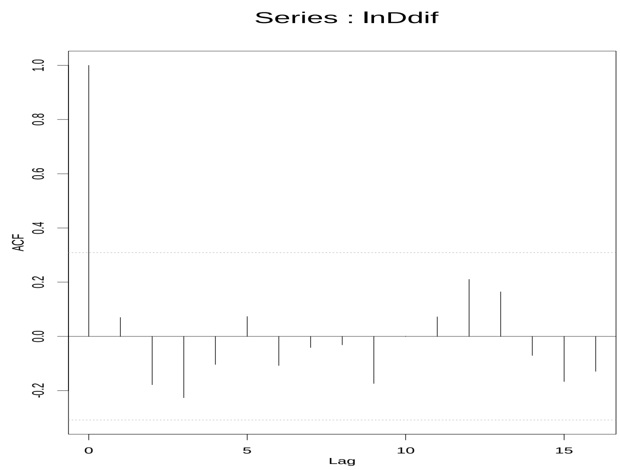

Figure 1.3: ACF plot of ϵ_t

- Costs associated with the production model are:
  - Cost of producing a plane to 50% or 100% completion state.
  - Cost of storing the plane in cell 1 and cell 2. These costs are dependent on the storage time.
- Revenue generated is *Sale Price* minus *Total Cost*, where the *Sale Price* is computed based on the delivery dates using the option pricing mechanism discussed in Section 1.5.
- Rate of production in the cells 1 and 2 has to closely follow the demand. Therefore the rates at which partially built planes are constructed was also modeled as an exponential function based on the input data.

An alpha version of a simulation was constructed to test our hypotheses. In order for our model to be successful, the simulation had to verify that our queuing strategy would generate more profit than Boeing's current strategy. Given the assumptions and the time frame, the simulation results were inconclusive. A better strategy for determining the rates at which planes are produced in cells 1 and 2 is needed for the simulation to be effective.

1.5 Using Option pricing to determine the worth of the airplane

In this section we will describe the option pricing mechanism used to determine the sale price of an airplane under the assumption that the option is a European option.

As was shown by Black, Scholes and Merton in 1973, the value c of a European call option with exercise price X and expiration date T is:

$$c = S_0 N(d_1) - X e^{-r_f T} N(d_2) \qquad (2)$$

where:

- S_0 is the spot price;

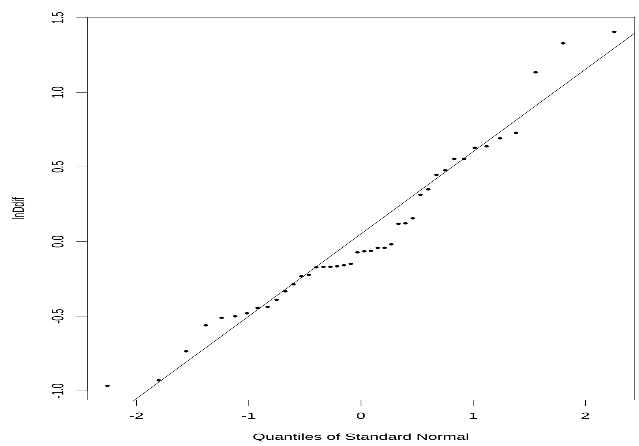

Figure 1.4: qq-plot of ϵ_t

- r_f is the interest rate;
- $d_1 = \frac{log\left(\frac{S_0}{X}\right)+\left(r_f+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$;
- $d_2 = d_1 - \sigma\sqrt{T}$;
- $N(d) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{d} e^{-\frac{x^2}{2}} dx.$

The option price for an airplane based on equation 2 is shown in Figure 1.4, where the *x-axis* is the time line and the point (x, t) is the cost of buying a plane t days from the current day.

The assumptions made in plotting this graph are:

- The plant never closes. This means that there is a continuous production.
- No employees are laid off.
- The spot price of one plane is 75 million dollars.
- The exercise price of the option is 75 million dollars.
- The risk free interest rate is $r_f = 0.03$.
- The price volatility is $\sigma^2 = 0.3$.

1.5.1 Real options

In Boeing's case, the option of buying an airplane is in fact a *real option*. A real option is the flexibility to make decisions about real assets, decisions which can involve adoption, abandonment, exchange of one asset for another or modification of the operating characteristics.

In our specific problem we have the following four real options:

- to maintain or to add excess capacity (how much should we add?);

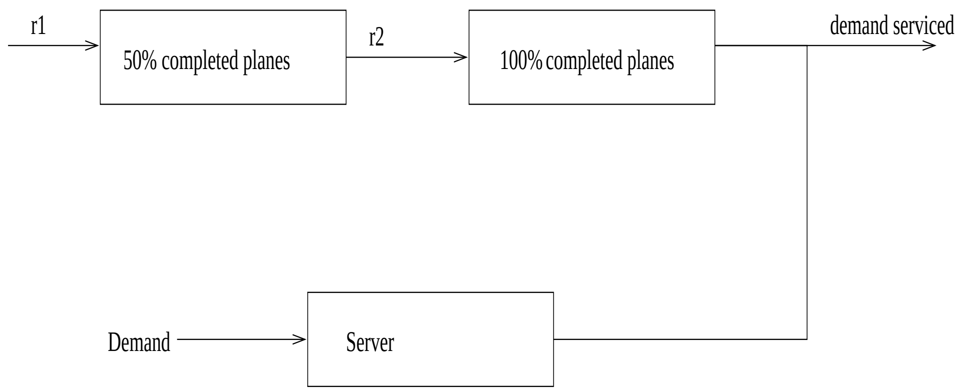

Figure 1.5: The queuing model

- to keep the plan operational;
- to mothball (we do not use the plant, but we still keep the people and machines inside it);
- to abandon, get rid of the plant.

For each of the real options outlined above we have to determine the option value and the exercise price. We adapt the technique of Brennan and Schwartz, outlined in the next section, to our domain.

The option price and the exercise date decisions depend on the structure of the stochastic demand. We will assume that the demand D follows the following diffusion process:

$$dD = \alpha\left(D,t\right)dt + \sigma\left(D,t\right)d\omega$$

where:

- α is the expected growth in demand, measured in orders per unit of time;
- σ is the annual standard deviation of underlying asset returns;
- dω is a Wiener process with zero drift and unit variance per unit time. That is, $\omega(t) - \omega(t-1)$ is normally distributed with expectation 0 and variance 1 and is independent of $\omega(\alpha) - \omega(\alpha - 1)$ for any time interval between $\alpha - 1$ and α that does not intersect the time interval between $t-1$ and t.

We first investigate the pure option to add capacity. Using the risk neutral interpretation (we look at expectation and ignore the risk premium without losing generality) and Itô's lemma we get the equation (similar to that in reference [1] obtained for the mine pricing model):

$$rW = W_D\left(\frac{E[dD]}{dt}\right) + \frac{1}{2}\sigma^2\left(D\right)W_{DD}$$

where:

- $W\left(D,t\right)$ is the value of the option to meet excess demand.

The left side of the above equation represents the required return per unit time of a risk-neutral investor for an investment in the option (see [5]). The right side represents the expected return to ownership of the option (per unit time), assuming that the underlying asset is priced by a risk-neutral investor (see the same as above)

The drift $\frac{1}{2}\sigma^2(D)\,W_{DD}$ is due to Itô's lemma.

The value of the option to abandon the plant plus the net cash flow will be now given by the equation:

$$\frac{1}{2}\sigma^2(D)\,\nu_{DD} + \left(\frac{E[dD]}{dt}\right)\nu_D - r\nu + pD = 0$$

where:

- pD represents the net profit (p is the net profit from the sale of one plane).

Using the information we obtained from Boeing's public website, and from our own time-series analysis, we actually found the values of $\sigma^2(D)$ and $\frac{E[dD]}{dt}$ and replaced them in the equations written above. In conclusion, these two second order Euler differential equations can now be solved.

For example, the general solution for the first equation is:

$$W(D) = \beta_1 D^{\gamma_1} + \beta_2 D^{\gamma_2}$$

where β_1 ,β_2 are easy to find and γ_1 ,γ_2 which will also appear in $\nu(D)$ can be determined from boundary conditions such as:

- value matching conditions:

$$W(D_L) = \nu(D_L) - E$$
$$\nu(D_H) = W(D_H) - I$$

  where D_L is the lowest demand, D_H is the highest demand, E is the value of abandonment and I is the value of investment

- optimality conditions:

$$W(D_H) = \nu(D_H)$$
$$W(D_L) = \nu(D_L).$$

In Boeing's case, we think it is a good idea to use also the mothball decision.

1.5.2 Valuation of the plant using the Brennan-Schwartz model

The approach outlined in this section is described by Brennan and Schwartz in reference [1]. Consider the value of an airplane production plant H. We assume that the price of the airplane produced at this plant is not subject to fluctuations. The total value of the plant output depends on the number of incoming orders D, which is a random variable following a Wiener process, as described in Section 1.5. For simplicity, we neglect all dependencies on other macroeconomic variables. This can be justified by observing that the rate of output is implicitly dependent on other factors, e.g., the current instantaneous riskless interest rate.

By Itô's lemma (as in [2]), the change in the value of the plant is given by

$$dH = H_D dD + H_t dt = \frac{1}{2}H_{DD}(dD)^2. \tag{1.1}$$

We neglect the impact of taxation, as well as the possibility of catastrophic macroeconomic events (crash of the stock market, war etc.). Therefore, the revenue delivered by the plant is given by $q(D - A)$, where q is the rate of production and A is the marginal cost of production incorporating the fixed costs. Applying the arbitrage argument as in [1], we arrive at the following equation for H:

$$\frac{1}{2}\sigma^2 D^2 H_{DD} + q(D - A) + H_t + (\rho S - C)H_D - \rho H = 0, \tag{1.2}$$

where σ is the volatility of demand, ρ the riskless interest rate, S the price of the airplane, C the convenience yield.

The boundary conditions can be set as usual, i.e.,

$$H(0) = F, H_{DDD}(P) = 0, \tag{1.3}$$

where F is the fixed cost of maintaining the plant and P is its production capacity. The terminal condition at time T when the order is filled (planes delivered) can be set as

$$H(T) = q(D - A). \tag{1.4}$$

Equation (1.2), subject to boundary conditions (1.3) and the terminal condition (1.4), can be solved by conventional numerical methods.

1.6 Conclusion

We showed that the nature of demand follows a geometric Brownian motion. This conclusion was reached by doing a time series analysis of the data. For simulation purposes the selling price of an airplane was determined by the Black Scholes model for the European options. We used the approach of Brennan and Schwartz to determine the valuation of the plant when the options are real.

In conclusion we state that, maintaining inventory might be a good idea to increase the revenue. Modeling of the projected demand and the option pricing for airplanes needs to be studied in more detail for the simulation to be effective. Valuations of the plant based on real options need to be explored further.

Preliminary results suggest that a detailed study is needed to evaluate the efficacy of the approach.

1.7 References

[1] M.S. Brennan and E.S. Schwartz, Evaluating Natural Resource Investments, The Journal of Business, 58 (2), 1985, 135 - 158.

[2] P. Wilmott, S. Howison, J. Dewynne, The mathematics of Financial Derivatives: A student introduction, Cambridge University Press, Cambridge, 1998.

[3] J. A. Buzacott and J. G. Shantikumar, Stochastic models of manufacturing systems, Prentice Hall, 1993.

[4] J. A. Buzacott and D. D. Yao, On queuing network models of flexible manufacturing systems, *Queuing Systems: Theory and Applications*, 1:5-27 1986.

[5] R. A. Jarrow, Finance Theory, Prentice Hall, 1986.

[6] J. W. Tukey, Exploratory Data Analysis, Addison Wesley, 1977.

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> with(finance):T:=1; exercise:=75; rate:=0.03;
> sdev:=0.07;
>
```

$$T := 1$$

$$exercise := 75$$

$$rate := .03$$

$$sdev := .07$$

```
> for i from 1 by 1 to 364 do t[i]:=i:nperiods:=1-i/365:
  amount:=exercise*exp(rate*nperiods):
> price[i]:=evalf(blackscholes(exercise, exercise, rate, nperiods,
  sdev)):
> od:
> l:=[[n,price[n]] $n=1..364]:
> plot(l,x=1..364);
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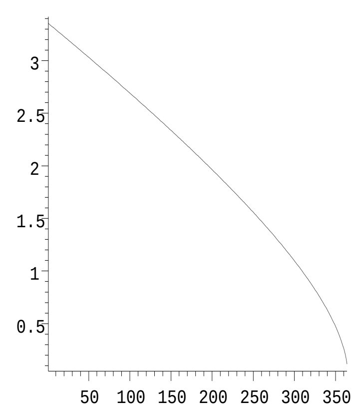

Figure 1.6: Option price for the planes