A jump diffusion model for spot electricity prices

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1 Introduction

At the outset, to appreciate the background of this project, we quote [3]:

"Over the last 10 years, major countries have been experiencing deregulation in generation and supply activities. One of the important consequences of this restructuring is that prices now determined according to the fundamental rule of supply and demand: there is a 'market pool' in which bids are placed by generators to sell electricity for the next day are compared to purchase orders".

Prior to that, the regulators used to set the price based on the cost of generation, transmission and distribution and the price to the consumer was essentially fixed for long period of time. A large fraction of the literature on electricity today belongs to the economics of deregulated electricity market from the perspective of the regulators. In the market mechanism now operating today, the price will be determined by the interaction of the purchase orders placed by the retailers against the pool prices.

The deregulation of the electricity market has also led to increased trading activities in both spot and related derivatives like forwards and options. The risk of spot-price has forced retailers to manage the risk of the spot price through various hedging mechanisms. Many retailers provide incentives to the consumers to enter into long term contract with predetermined price structure, but that still leaves the risk of buying price. It is in this context modelling the stochastic behaviour of the spot price of electricity has become important.

One feature of the electricity market that is unique to this commodity is that electricity is not storable, although, it may be argued that the concept of storability applies to hydro electricity generation. Since, in general, it cannot be stored the spot price is likely to be determined by the spot concerns, e.g., spot demand and supply constraints. The ability to store any commodity has the effect of smoothing the evolution of the spot price to some extent. As a result of its absence, price spikes are a regular feature of the electricity spot prices in most countries that have deregulated this market. Price spikes are possibly due to disruption in transmission, unscheduled outages, extreme weather changes or a combination of all these events. Additional details about the characteristics of this market may be found in [3].

We will now review some of the salient characteristics of the electricity prices in the deregulated market. In standard commodity-futures markets the concept of convenience yield plays a key role in the relationship between the spot and the forward prices. The convenience yield is a way of expressing the fact that an investor is sure of available supply when the demand for using that commodity arises at a future date. The non-storability of electricity makes the concept of convenience yield difficult to apply. This implies that the spot price itself should contain all the characteristics of the price process that would be necessary to impute prices of derivatives contracts written on electricity prices.

Next, we outline the important temporal characteristics of spot electricity prices observed in most markets. A detailed description of these characteristics may be found in [3]. Mean reversion is an important feature of spot electricity prices. The prices tend to fluctuate around values determined by cost of production and the level of demand. The mean reversion level may be constant or periodic with a trend. Seasonality is another obvious characteristic. The prices change by time of day, week, month and the year in response to cyclical fluctuations in demand. Another feature already mentioned before is that of price jumps or spikes. A point to note is that technically price does not jump to a new level (to stay there) but spikes and quickly reverts to their previous levels. This price spike has been the most difficult aspect to model appropriately.

It is, therefore, clear from the above discussions that a pure diffusion process would not adequately capture the characteristics for electricity price series. A pure diffusion process approach has worked well in stock price modelling. For the electricity market, however, we need to incorporate a jump component with an appropriate intensity function to capture the spikes. Many of the traditional modelling approaches applied to financial market data e.g. equity, foreign exchange, and interest rates etc do not work well with spot electricity prices. This has been the experience for most researchers in this area as discussed in [3]. With respect to the equity market though, the work by [5] is an important contribution to detect jumps (as opposed to spikes). Their focus has been whether jump risks in stock returns are diversifiable.

In this paper we attempt to combine the ideas expressed in the cited literatures and explore a jump diffusion model for spot electricity prices in NSW. We allow both a deterministic time dependent factor as well as a latent factor combined with Poisson jumps to capture the observed characteristics of the spot electricity price series. We show how to calibrate such a model to the market data and describe the appropriate algorithm for that. The algorithm we employ generates, in a natural way, one period ahead forecast of spot electricity price. This, in turn, helps us determine the "goodness of fit" of such a model.

2   A Model for Spot Electricity Prices

In modelling commodity prices the approach of [7] has become quite popular. Their analyses depend upon both short dated and long dated futures contracts of the commodity. It also relates to the convenience yield as normally applied to futures contracts. Since the electricity, as a commodity, is different in this respect due to non-storability of the commodity for possible future consumption, the short-term, long-term concept introduced by [7] may not strictly apply to this market. Nevertheless, the ideas contained in [7] have important bearing in dealing with the electricity market.

It is clear from the earlier discussions that price jumps or price spikes are a natural characteristic of the electricity market and have to be built into the model. It is also useful as we may be able to adopt the models we develop here for pricing derivatives contracts on the electricity spot prices. To reliably model contingent claims prices we have to incorporate jumps in addition to the usual diffusion assumptions in the price process, which makes it far more complex compared to pricing derivatives on equities. In this context we need to be mindful of the theorem by [2] that leads to closed form solution, in most cases, of the contingent claims when the underlying security follows an affine[1] jump-diffusion process (AJD). Although, we are not strictly focusing on electricity derivatives contract in this paper, we will strive to stay

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[1] Affine structure implies linear dependence on state variables

close to the AJD process so that our approach can be easily adapted for contingent claims pricing later.

Many researchers traditionally model log of the spot price of the commodity as in [7]. The existence of a significant jump component in the electricity prices it is worthwhile to re-consider whether a logarithmic transformation is useful. The logarithmic transformation affects the estimation of the jump component due its effect on the skewness of the distribution of the series. Since the derivatives contracts are written on spot price level and not on its log transformation, developing models of log transformation of spot price will not be useful. [6] find that the model of price level fits the forward contract prices better than the log-price level. In this paper we will, therefore, model the price level and not its log transformation. That way the models we develop will be better suited to pricing derivatives contracts on spot electricity prices.

In the original approach of [7] the log of the commodity price is modelled via two factors, both unobserved. The first factor captures the short-term variations and is modelled by an Ornstein-Uhlenbeck (OU) process whereas the second factor (the long-term variations) is modelled by an Arithmetic Brownian process (ABM). The commodity examined in Schwartz and Smith is crude oil and it displays non-stationarity. Hence the inclusion of the ABM process in their analysis is not only meaningful but is also a necessity since the OU process alone would not be able to capture the dynamics. Since our spot electricity price series is stationary (found by Augmented Dickey-Fuller tests) we need only include the OU process to capture the dynamics without the jumps. To capture the jump characteristic we include a jump component in the OU process.

Another difference from the structure in [7] for electricity spot prices is the inclusion of a time-dependent, deterministic function to capture the observed seasonality in the series. This arises mainly due to the nature of household consumptions of electricity depending on the season we are in. This also indicates that the intensity process for the Poisson component capturing the jumps in the series may not be constant, and is more likely to depend on seasonal factors.

With this background we are in a position to specify the spot price process (P_t) mathematically in terms of a deterministic, time-dependent function $f(t)$, and a state variable X_t. Although many researchers specify their models in continuous time setting and for implementation purposes use Euler discretisation, we prefer to stay in the discrete framework from the start. We set daily average electricity spot price, measured in dollars per megawatt-hour,

$$P_t = f(t) + X_t. \tag{1}$$

The deterministic, time-dependent part is described as a sinusoidal function along with a weekday dummy variable (wkd_t). This specification is similar to that used in [6]. This last variable is to help distinguish between the price on a weekday and a weekend. We use the following specification for $f(t)$:

$$f(t) = \beta_0 + \beta_1 t + \beta_2 \sin\left\{(t+\beta_3)\frac{2\pi}{365}\right\} + \beta_4 \sin\left\{(t+\beta_5)\frac{4\pi}{365}\right\} + \beta_6 \text{wkd}_t, \tag{2}$$

where $\text{wkd}_t = 1$ if the day is a weekday otherwise it is zero.

The OU component X_t describes the unobserved component that captures the short-term dynamics, allows occasional jumps and is characterised by volatility clustering that is common to most financial time series. The notion of unobserved component is in the sense of state

space models and filtering theory. This volatility clustering is modelled as a GARCH (1,1) process. Thus we specify:

$$X_t = \phi X_{t-1} + h_t^{0.5}\varepsilon_{1,t} + J(\mu_J, \sigma_J)\Delta\Pi(\lambda_t), \tag{3}$$

where $\varepsilon_{1,t} \sim N(0,1)$, the time varying variance $h_t = \alpha_0 + \alpha_1\varepsilon_{1,t-1}^2 + \alpha_2 h_{t-1}$. The jump component is controlled by a Poisson-distributed variable with time dependent intensity function λ_t, and the jump amplitude is governed by a normally distributed variable with mean μ_J and variance σ_J. To capture seasonal effects in the jump component, we set

$$\lambda_t = \gamma_1\text{winter}_t + \gamma_2\text{autumn}_t + \gamma_3\text{spring}_t + \gamma_4\text{summer}_t \tag{4}$$

where the seasonal dummy variables indicate whether a particular date is in one of these seasons.

The specifications in equations (2) and (4) are just an assumption of the form of these functions for the dataset being analysed. These are based upon basic analysis of the data and similar decisions taken in the literature already referred to above. In this article we have not explored whether other forms of these functions have better properties.

The set of equations (1) and (3) describe our modelling approach to electricity spot prices and is already in state space form. In this state space representation of our problem equation (1) is the measurement equation and equation (3) is the state transition equation. The number of unknown parameters in this model is seventeen and these are estimated by the maximum likelihood method. Due to the presence of the unobserved component, X_t , we resort to Kalman filter to develop the likelihood function recursively. This process is described in detail, along with the modification needed due to the jump process, in the appendix. The parameter set is conveniently given by the vector,

$$\Theta \equiv \{\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \alpha_0, \alpha_1, \alpha_2, \mu_J, \sigma_J, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \phi\}. \tag{5}$$

There are seventeen parameters to be estimated and it is not an easy optimisation task.

Although we have described in the appendix in detail the steps of the filtering algorithm including the modification needed to accommodate Poisson jumps, additional insights may be gained from Chapter 6 in [4]. In order to allow GARCH variance in the state dynamics, we need further non-trivial modification to the standard Kalman filter [4], and in particular Chapter 6 is an excellent reference source for this topic. Thus to conserve space, we refer the reader to that source material.

We implemented this algorithm in Gauss and used numerical optimisation to estimate the parameters as well as the standard errors from the information matrix. The filter algorithm produces the one-step ahead prediction of the state vector, which in this case is X_t.

3   Data and empirical results

Data used for this study was provided by Integral Energy. This represents every half hour NSW pool price covering the period 2002 through to 2006. We, however, use the daily average price for our modelling. We also estimate the model for two different periods. The first sample covers 2002 - 2003 containing 730 observations and the second sample covers 2004-2006 containing 1096 observations. Daily average prices are expressed in Australian dollars.

As a first step we check the stationarity of the spot price series using an ADF test. The main focus here is to statistically reject the unit root hypothesis in the electricity spot price series for entire sample period examined. The ADF test is based upon the following equation, where P_t is the daily average electricity spot price and Δ is the difference operator:

$$\Delta P_t = c + \gamma P_{t-1} + \sum_{i=1}^{n} \delta_i \Delta P_{t-i} + \eta_t. \tag{6}$$

The hypothesis being tested is $H_0 : \gamma = 0$ as opposed to $H_1 : \gamma < 0$. The quantity n in the above test is decided by a sample specific check so that the residual series is uncorrelated. The t-statistic for this test for the sample of 2002-2003 is -13.52 and that for the sample 2004-2006 is -27.04. The critical values for this test are obtained from the econometric software Eviews and the existence of unit root is rejected for both samples.

This unit root test convincingly supports the view that we need only the OU component in our model as opposed to [7] where both an OU and an ABM component were needed. For our analysis this OU component is given by the equation (3).

Focusing on the parameter estimates in Table 1, we note that the time trend component in the deterministic part (equation (2)) is insignificant in the second sample (2004-2006), whereas in the first component it is highly significant and displays downward bias. Although we cannot draw any firm economic conclusion from this, it is worthwhile to keep in mind that this market is still evolving and maturing.

Both samples display heteroscedasticity as seen from the quantity $(\alpha_1 + \alpha_2)$. The persistence in volatility in the short-term component (the OU part) has, however, gone down in the second sample. It is also interesting to observe that the autoregressive parameter (ϕ) of the unobserved component in the second sample is about half the size of the earlier sample. It may stem from more efficient pricing by the participants in this market and is probably the result of better understanding of this commodity in this evolving market.

We now focus on the jump component, i.e., the parameters (μ_J, σ_J). Both these parameters are highly significant in both samples. Although the mean jump amplitude is higher in the second sample, its volatility is an order of magnitude higher in the second sample. That it might be so is also apparent from Figures 1 and 3 which are plots of the electricity spot prices over the whole periods. This may result from the supply concerns in the second sample period or the reflection of uncertainty in the regulatory environment governing this market. In this paper we are not in a position to shed further light on this aspect of the results. The appropriateness of the time varying jump intensity as captured by equation (4) is supported by the statistical significances of the estimated parameters in both sample periods.

Finally, we check on the predictive power of the model in both sample periods. Since the filtering algorithm recursively produces one step ahead projection of the state variable, we have shown in Figures 2 and 4 the possible price paths, i.e., the expected electricity spot prices. Using this information and the subsequently realised prices we can make comparative judgement about the usefulness of the model. In the traditional statistical sense the forecast ability of a model is judged by some measure of association between the forecasts and the realizations. However, there is an alternative to R-square measures and this is given by Theil's Inequality Coefficient. It was originally proposed in 1961 and has been employed by several researchers since then. We have applied Theil's Inequality Coefficient (TIC) to test this performance.

The TIC (Theil's Inequality Coefficient) is given by the following expression. We assume

that the variable of interest is z_t for $t = 1, 2, ...T$, and its estimated value is given by $\hat{z}_t$:

$$\text{TIC} = \frac{\sqrt{1/T \sum_{t=1}^{T}(z_t - \hat{z}_t)^2}}{\sqrt{1/T \sum_{t=1}^{T} z_t^2} + \sqrt{1/T \sum_{t=1}^{T} \hat{z}_t^2}}. \tag{7}$$

This coefficient always lies between zero and one, where the smaller the coefficient the better the estimate. For additional application of this measure the readers may refer to [1].

The computed value of TIC is smaller in the first sample compared to that in the second sample. Both values, however, indicate reasonable success in capturing the price path one day ahead. The lower value of the autoregressive parameter (ϕ) in the second sample may have contributed to lower predictive power in the second sample.

4   Summary and conclusion

We have explored the modelling[2] of the electricity spot prices in New South Wales through a jump-diffusion process mixed with time varying deterministic trend component. The unobserved factor is driven by a diffusion process with time varying variance and a Poisson distributed jump component. We have outlined the algorithm needed to extract this latent factor from the observed price series and find that its one-step ahead prediction does contribute to the forecast of a complex electricity spot price series.

The very nature of the likelihood function of the Poisson mixture of Gaussian distribution requires some approximation for implementation purposes. To keep the computation burden low we have kept the upper limit of the infinite series summation to a small value. This is consistent with published articles in this area of research. One way to extend this study would be to experiment with this upper limit to check whether it improves predictive accuracy. Although we have allowed the jump amplitude to be a normally distributed variable, there may be other distributions that could prove useful in improving the predictive accuracy. This remains another possible extension of this study.

Standard errors are in parentheses below the parameters. TIC represents Theil's inequality coefficient, a measure of model's ability to predict the observed data. This is described in the text.

Appendix

State space model (SSM) with Poisson jumps and Kalman filter
    The SSM in its basic form retains a VAR (1) structure for the state equation,

$$y_t = \Gamma y_{t-1} + w_t + \zeta_t \tag{8}$$

where the state equation determines the rule for generation of the states $y_t, p \times 1$ vector, from the past states y_{t-1}, for all time points $t = 1, 2, ..., T$. For completeness we assume that w_t are $p \times 1$ independent and identically distributed zero-mean normal vectors with covariance Q_t. We assume that Q_t is diagonal and may be constant. When we allow GARCH effect

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[2]In the published research in electricity spot price modelling, jump-diffusion model has emerged as the main technique. In the absence any other viable approaches it is not practical to compare the forecast ability of our model with a competing model as yet.

	2002-2003	2004-2006
β_0	11.88430	1.02417
	(3.4316)	(0.0070)
β_1	-7.19647	0.00953
	(1.4750)	(0.0078)
β_2	0.91000	1.01030
	(0.0260)	(0.0052)
β_3	1.03455	1.01031
	(0.0152)	(0.0052)
β_4	0.81608	1.01055
	(0.0423)	(0.0052)
β_5	1.06108	1.01057
	(0.0159)	(0.0052)
β_6	1.30011	1.04473
	(1.5468)	(0.0122)
α_0	0.11923	1.05235
	(0.0559)	(0.0064)
α_1	0.55549	0.33955
	(0.0075)	(0.0013)
α_2	0.25873	0.33867
	(0.0040)	(0.0013)
μ_J	11.12589	16.18870
	(1.2141)	(0.9561)
σ_J	240.36013	1333.91930
	(13.0915)	(57.0935)
γ_1	1.97147	2.14528
	(0.1567)	(0.1322)
γ_2	1.92343	1.88024
	(0.1460)	(0.1174)
γ_3	2.48438	1.94322
	(0.1686)	(0.0306)
γ_4	1.84487	2.07050
	(0.1436)	(0.0381)
ϕ	0.16446	0.08159
	(0.0265)	(0.0218)
TIC	0.3839	0.5489

Table 1: Maximum Likelihood Estimates of the Parameters of the Jump Diffusion Model

in some of the elements of the state vector, the corresponding element in Q_t would then be time varying. The noise term ζ_t introduces the jump in the process and is assumed to be represented by

$$\zeta_t \sim N(j \cdot \mu_J, j \cdot \sigma_j^2), j = 1, 2, ..., \infty. \tag{9}$$

In equation (2) j is a Poisson distributed random variable during a small interval Δt characterised by a single parameter $\lambda_t \Delta t$. The state process is assumed to have started with the initial value given by the vector, y_0, taken from normally distributed variables with mean vector μ_0 and the $p \times p$ covariance matrix, Σ_0.

The state vector itself is not observed but some transformation of these is observed but in a linearly added noisy environment. Thus, the measurement equation is given by,

$$z_t = d_t + A_t y_t + v_t. \tag{10}$$

In this sense, the $q \times 1$ vector z_t is observed through the $q \times p$ measurement matrix A_t together with the $q \times 1$ Gaussian white noise v_t, with the covariance matrix, R. In equation (3) d_t is a purely deterministic time dependent variable. We also assume that the two noise sources in the state and the measurement equations are uncorrelated.

The next step is to make use of the Gaussian assumptions and the independence of Poisson distributed events and across times and produce estimates of the underlying unobserved state vector given the measurements up to a particular point in time. In other words, we would like to find out, $E(y_t | \{z_{t-1}, z_{t-2}, \cdots, z_1\})$ and the covariance matrix, $P_{t|t-1} = E[(y_t - y_{t|t-1})(y_t - y_{t|t-1})']$. This is achieved by using Kalman filter and the basic system of equations is described below.

Given the initial conditions $y_{0|0} = \mu_0$, and $P_{0|0} = \Sigma_0$, for observations made at time $1, 2, 3...T$,

$$y_{t|t-1}^{(j)} = \Gamma y_{t-1|t-1}^{(j)} \tag{11}$$

$$P_{t|t-1}^{(j)} = \Gamma P_{t-1|t-1}^{(j)} \Gamma' + Q_t \tag{12}$$

$$y_{t|t}^{(j)} = y_{t|t-1}^{(j)} + K_t^{(j)}(z_t - A_t y_{t|t-1}^{(j)}), \tag{13}$$

where the Kalman gain matrix

$$K_t^{(j)} = P_{t|t-1}^{(j)} A_t'[A_t P_{t|t-1} A_t' + R]^{-1} \tag{14}$$

and the covariance matrix $P_{t|t}$ after the tth measurement has been made is

$$P_{t|t}^{(j)} = [I - K_t^{(j)} A_t] P_{t|t-1}^{(j)}. \tag{15}$$

Equation (4) forecasts the state vector for the next period given the current state vector and the Poisson jump. Using this one step ahead forecast of the state vector it is possible to define the innovation vector as

$$\nu_t^{(j)} = z_t - A_t y_{t|t-1}^{(j)} \tag{16}$$

and its covariance as

$$\Sigma_t^{(j)} = A_t P_{t|t-1}^{(j)} A_t' + R. \tag{17}$$

The description of the above filtering algorithms assumes that the parameters are known. In fact, we want to determine these parameters and this is achieved by maximizing the

innovation form of the likelihood function. The one step ahead innovation and its covariance matrix are defined by the equations (9) and (10) and since these are assumed to be independent and conditionally Gaussian, the log likelihood of the Poisson mixture of normal distribution is given by

$$
\text{Ln}(L) = \sum_{t=1}^{T} Ln \left[\sum_{j=0}^{\infty} \omega(\lambda_t, j) \right.
$$
$$
\left. \left\{ (2\pi)^{\frac{-q}{2}} \cdot |\Sigma_t(\Theta)|^{\frac{-1}{2}} \exp \left(-\frac{1}{2} \nu_t^{(j)'}(\Theta) \Sigma_t^{(j)-1}(\Theta) v_t^{(j)}(\Theta) \right) \right\} \right]. \tag{18}
$$

In this expression Θ (collection of all the unknown parameters) is specifically used to emphasize the dependence of the log likelihood function on the parameters of the model. Once the function is maximized with respect to the parameters of the model, the inferred state vector is also available.

In practice the infinite sum in the above log likelihood function has to be approximated by something more appropriate for computation. The published papers in this area using equity market data normally use 10 as an upper limit for the summation term. For example, Kim, Oh and Brooks [5] use 4 for their study of jump risks in equity return. In this paper we use 6. Higher value will of course give better approximation but at the expense of rapidly increased computation time.

There are different numerical approaches that may be taken to carry out the maximization of the log likelihood function. The computational complexity and other numerical issues are beyond the scope of this paper.

References

[1] Bali, T. G. & Weinbaum, D. (2007) A conditional extreme value volatility estimator based on high-frequency returns, *J. Econ. Dyn. Cont.*, **31**, 61-397.

[2] Duffie, D., Pan, J. & Singleton, K. (2000) Transform Analysis and Asset Pricing for Affine Jump-Diffusions, *Econometrica*, **68**, 1343-1376.

[3] Geman, H. & Roncoroni, A. (2006) Understanding the Fine Structure of Electricity Prices, *J. Bus.*, **79**, 1225-1261.

[4] Kim, C-J., & Nelson, C. R. (1999) *State Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications* The MIT Press, Cambridge.

[5] Kim, M-J., Oh, Y-H. & Brooks, R. (1994) Are Jumps in Stock Returns Diversifiable? Evidence and Implications for Option Pricing, J. Fin. Quant. Anal. **29**, 609-631.

[6] Lucia, J. & Schwartz, E. (2002), Electricity Prices and Power Derivatives: Evidence from Nordic Power Exchange, Rev. Deriv. Res. **5**, 5-50.

[7] Schwartz, E. & Smith, J. E. (2000), Short-term Variations and Long-term Dynamics in Commodity Prices, *Manag. Sc.,* **46**, 893-911.