Chapter 6

Dynamics of Large Mining Excavators

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6.1 Introduction

Operators of large mining excavators could improve their efficiency if they were provided with a real-time knowledge of both the mass in the bucket and the force on the digging teeth of the machine. Although shaped roughly like a standard construction excavator, mining excavators are many times larger. Their payload can be over 40 tonnes, with the hydraulic cylinders operating at up to 30 MPa. Obviously, the intuition and feel that an operator may have for a smaller machine is lost on these goliaths, and any information about the forces on the bucket is important to the operator. During normal operation, one truck is loaded on one side of the machine while a second truck moves into position on the other side. The machine loads vehicles continuously, spending approximately one minute with each truck. Hence, to be of use to the operator, the payload and digging force must be obtained during normal operation of the machine with at most a one second delay.

To be economically viable, a device to measure the payload and digging force should be applicable to any excavator without extensive modelling and fitting costs and should be constructed of robust, low cost sensors limited to measurement of angles between the components of the excavator and pressures in the hydraulic cylinders. One previous approach to the problem was to develop a detailed model of the dynamic behaviour of the machine's arm and bucket. In practice, however, it is not economical to fit such a detailed model to each machine. Further, there are a large number of unknown effects from unmodelled dynamics, such as friction and hydraulic elasticity, and from errors due to sensor limitations and background noise in pressure and angle data.

The ideal device would not require technical fitting or modelling of each machine, but would be a straightforward mechanical installation, with a single device suitable for any machine. The idea that a *black box* be installed and *trained* to each machine leads naturally to the use of parametric or non-parametric regression techniques to estimate the functional form of the dependence of the payload on the configuration of the machine. A non-parametric model assumes only that the payload can be expressed as a function of the independent variables, and that this function can be approximated by a weighted sum of a set of basis functions. Linear regression is used with a set of training data to estimate the weights. In contrast, a parametric model assumes that the payload can be expressed as a known function of the independent variables and several parameters. The training data is then used to estimate the parameters.

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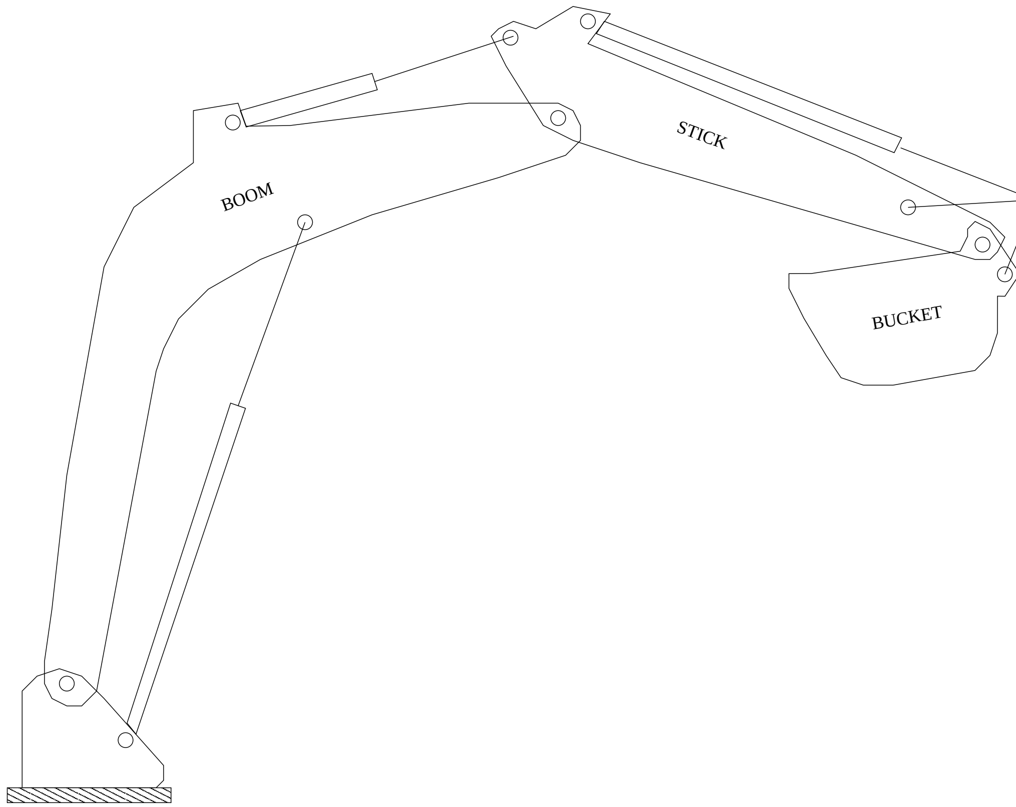


Figure 6.1: A typical excavator arm.

In this report we compare the use of parametric and non-parametric regression models and their suitability to the problem of determining the mass in the bucket. The problem of determining the force on the teeth of the bucket can be modelled in a similar fashion. However, it is not clear that there is a single force acting on the teeth of the bucket during digging and not a more complex contact between the bucket and the ground. For this reason, this report focuses on the problem of determining the payload mass. The problem of measuring the digging force on the bucket teeth is not dealt with in any detail.

The investigation of the non-parametric regression model indicates that the black box approach is impractical due to the large amount of training data needed to estimate the payload function. It would seem that both extremes, the detailed model on one hand, and the black box on the other, require too much time and effort to measure parameters to be of practical use. The parametric model requires far less data for training than the non-parametric regression model, and even though a detailed model of each type of machine must be developed, an exact measurement of the model parameters is not required. Hence, the parametric model appears to be the best approach.

6.2 Modelling approach

Figure 6.1 shows a typical excavator consisting of a boom, stick and bucket connected by pin joints and hydraulic cylinders. Using techniques from dynamics and robotics, the mass in the bucket can be specified as a function of the pressures in the hydraulic cylinders, the angles between each component and their derivatives (see Section 6.3). Hence, if all the parameters in this model could be determined, then determining the mass in the bucket would be a simple matter of evaluating the function given a time history of the angles and pressures. Unfortunately, this approach is impractical for several reasons. Obviously, even with a detailed model there are unmodelled features, such as friction in the pins and cylinders, and the hydraulic elasticity and inertia of the cylinders. There is also a large amount of noise in the data, particularly in the measurement of angular accelerations. However, the

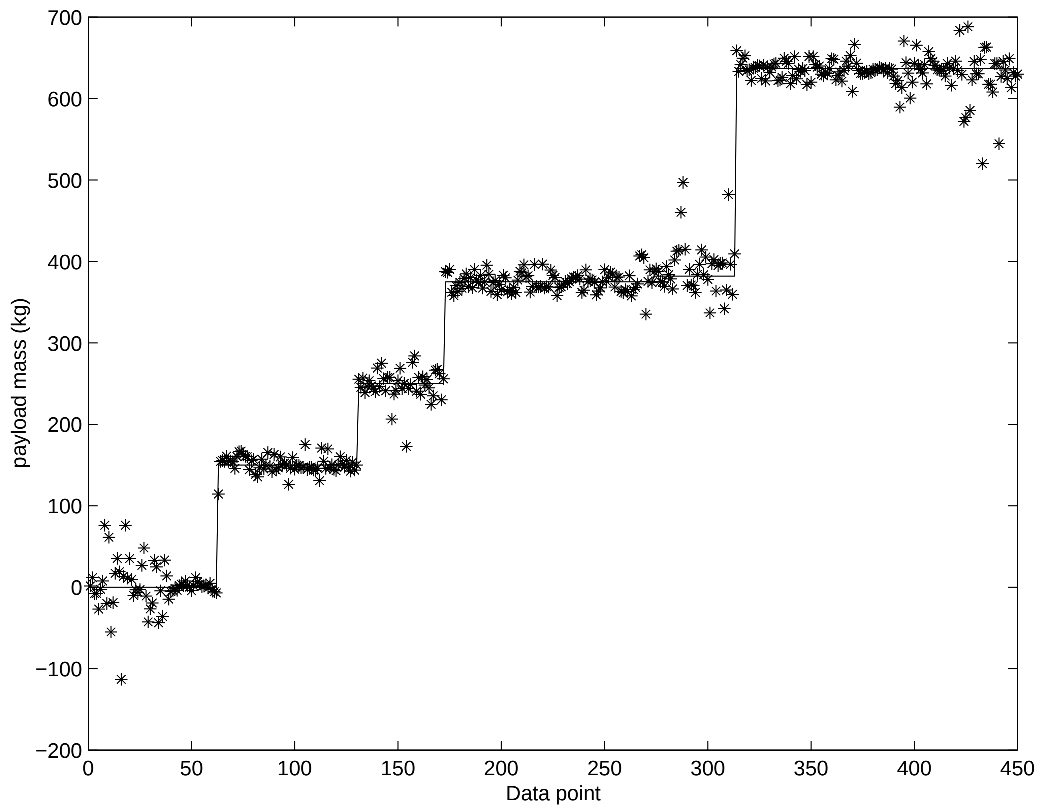


Figure 6.2: Approximation of the payload using radial basis functions. Evaluation of the function at the training points.

most significant drawback using a detailed model is the impracticality of measuring all the parameters for each machine. To be economically viable, a device to measure the payload should not require detailed measurements for each machine.

Another approach, diametrically opposed to the detailed model is to use non-parametric regression to estimate the payload function. Routines implementing these techniques are widely available. In fact, a simple search of the Internet revealed a set of public domain Matlab routines[7] for the approximation of functions using radial basis functions [2]. Briefly, this approach assumes that the function in question can be approximated by a weighted sum of radial basis functions and uses linear regression to determine the weights. Some sophistication is added to determine the most suitable set of basis functions. To test this method, data made available from RSI Technologies was used to construct an approximation to the payload function. The data was obtained by placing known masses in the bucket of an excavator and measuring the pressures and angles as the configuration of the arm was varied. To reduce the complexity of the problem, the testing was done only on a subset of this data for which the configuration was static. In this case, the payload function depends on the three angles describing the configuration of the arm and the pressure in one cylinder, but not on the angular velocities and accelerations. Figure 6.2 shows the plot of the approximated function for the data points used in the regression. To test the ability of the approximation to interpolate, data for a mass of 275kg was reserved and the model was trained on the remaining data. The resulting payload function was then evaluated for the 275kg data. The results are shown graphically in Figure 6.3. The method is clearly unable to interpolate between masses in the training data. Since the device is expected to be used for a wide range of payload masses, it is impractical to train the device to the level of detail required. We conclude, therefore, that the black box approach of non-parametric regression is not an economically feasible option.

[7] http://www.anc.ed.ac.uk/~mjo/rbf.html

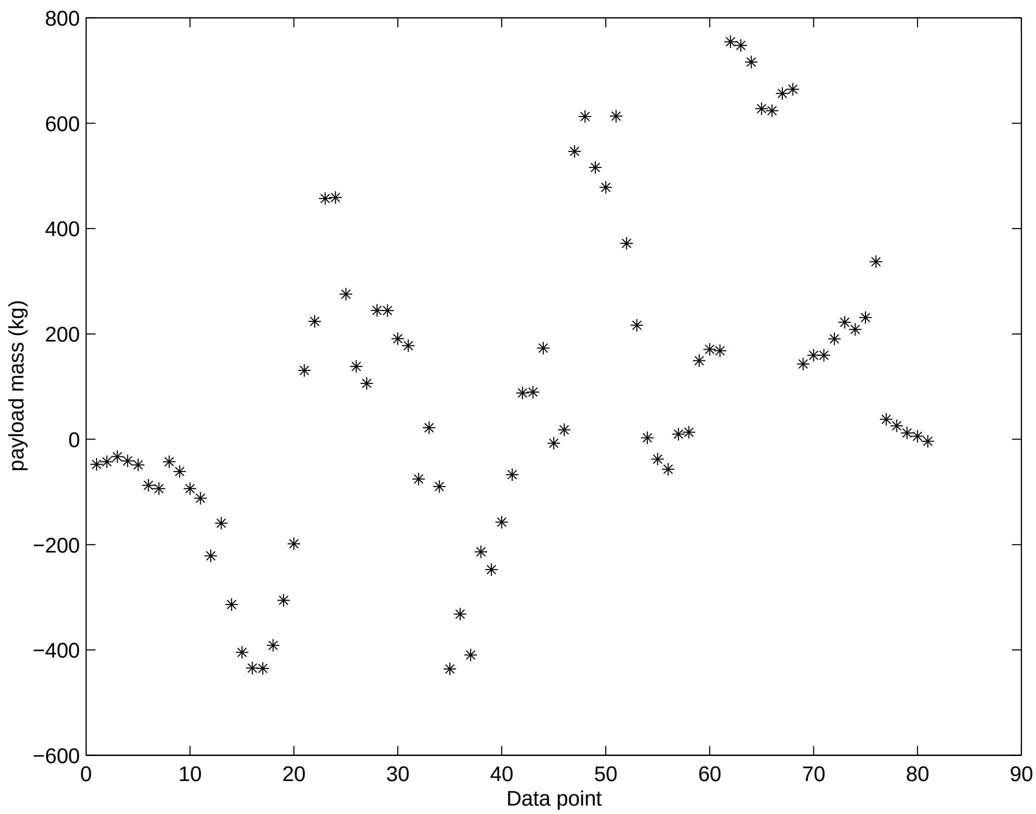


Figure 6.3: Approximation of the payload using radial basis functions. Evaluation of the function for masses not used in training.

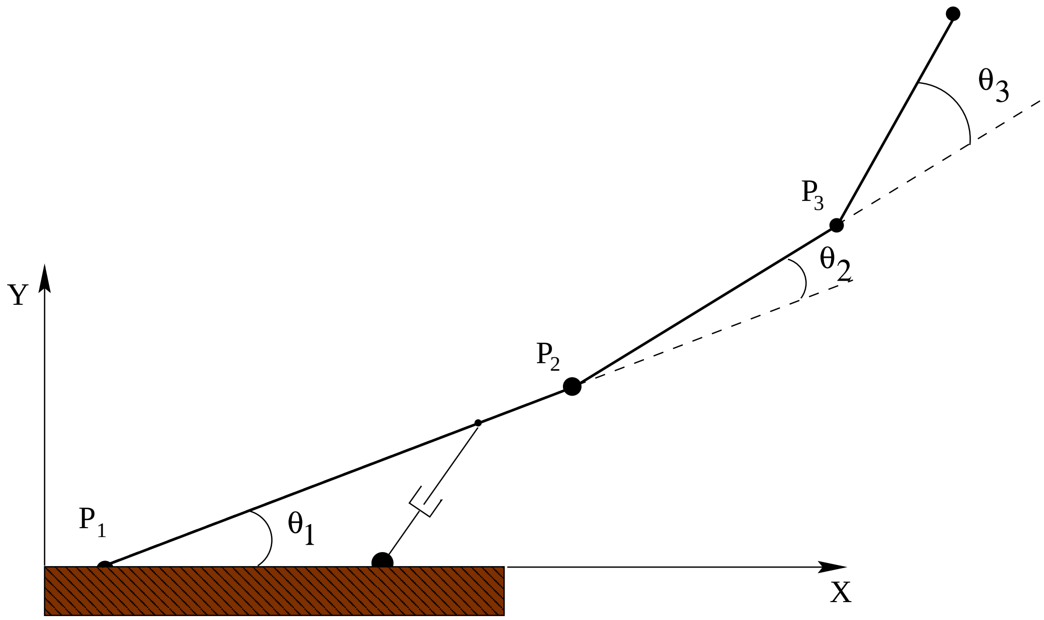


Figure 6.4: Coordinate system for the general model.

The poor ability of the non-parametric approach derives from the local nature of the approximation. In order to interpolate new masses not in the training data, some knowledge of the global form of the function is required. In the following section we develop a detailed model for an excavator arm. However, rather than measure parameters directly from the machine, we propose to use regression analysis of data for known weights to estimate the parameters in the model. In general, there is a non-linear dependence of the payload mass on the independent variables and the unknown parameters. To avoid the complications of using non-linear regression algorithms, the ideas are illustrated in Section 6.4 using a simple one-armed digger. For this model digger, the payload is a linear function of two parameters, allowing linear regression to be used for their estimation.

6.3 Model Excavator

The excavator shown in Figure 6.1 is generalized to the three link arm shown in Figure 6.4. A dynamic model of this generic excavator is developed using the Lagrangian approach. Each component of the digger is replaced by a single link. The links are connected by pins which allow each link a single degree of freedom and constrain all the links to lie in the same plane. The links are numbered from 1 through 3, with θ_1 being the angle of the first link with the horizontal plane and θ_i, $i > 1$, being the relative angle between links $i-1$ and i. The angle θ_0 is reserved for the rotation of the base around the Y axis.

Lagrange's equations of motion [1] specify that the energies satisfy the following system of equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = Q_i, \qquad i = 0, \ldots, n, \tag{6.1}$$

where Q_i is the rate of work done on the system by the non-conservative forces with increases in the angle θ_i. The digger arm is subject to conservative forces, due to gravity, and non-conservative forces through the hydraulic cylinders between each link and the torque applied to the rotating base.

A coordinate system is attached to the fixed base of the first link. The X axis is aligned with the digger arm, the Y axis vertical to the base and the Z axis perpendicular to the arm in the horizontal plane. A translated coordinate system for link i is attached to the pin connecting link $i-1$ to link i. P_i is the position vector of the i^{th} link in the fixed coordinate system.

Each link is a rigid body undergoing rotation and translation. Link i rotates with angular velocity $\omega_i = (0, \dot{\theta}_0, \sum_{j=1}^{i} \dot{\theta}_j)$ and it's base (the point P_i) translates with velocity v_i, where

$$
\begin{aligned}
v_1 &= 0, \\
v_i &= v_{i-1} + (P_i - P_{i-1}) \times \omega_{i-1}.
\end{aligned}
$$

The kinetic and potential energy of each link depends on the mass m_i, centre of mass r_i and inertial tensor I_i. For convenience, the centre of mass and inertial tensors are taken relative to position P_i for each link. Obviously, since the link is rotating rigidly in this coordinate system, r_i and I_i depend in a known fashion on the link angles θ_1 through θ_i. The kinetic and potential energy of each link are given by the expressions

$$T_i = \frac{1}{2} m_i (v_i \cdot v_i) + m_i v_i \cdot (\omega_i \times r_i) + \frac{1}{2} \omega_i I_i \omega_i^t, \tag{6.2}$$

$$V_i = m_i (P_i + r_i) \cdot (0, g, 0), \tag{6.3}$$

where g is the acceleration due to gravity. That is, T_i and V_i are the kinetic and potential energies of link i and $L = \sum_{i=1}^{3} T_i - V_i$ is the total energy use in Equation 6.1.

Work is done by the non-conservative forces as the lengths of the hydraulic cylinders change. Let $h_i(\theta_i)$ be the length of the cylinder connecting link i with the previous link, or the base in the case of the first link. Since each cylinder connects neighbouring links, the work corresponding to changes in θ_i depends only on the function $h_i(\theta_i)$ and the forces on the i^{th} cylinder as follows

$$Q_i = F_i h_i'(\theta_i). \tag{6.4}$$

The force F_i is proportional to the pressure difference across the cylinder minus any frictional forces. It is hoped that these frictional forces are small enough to be neglected. However, in the event that they are not, they may be assumed to be in proportion to the velocity of the cylinder $h'(\theta_i)\dot{\theta}_i$.

The excavator bucket is represented as the third link. Hence the payload mass is m_3. Since the potential energy of the final link, P_i, depends on each of the angles, the payload mass appears in equation in the system given by Equation 6.1. That is, any one of these equations can be used to derive a function for the payload in terms of the angles, cylinder pressures and parameters of the system. Note, however, that each equation depends only on a single pressure. Hence, for the non-parametric approach, the payload function can be assumed to depend on the three angles, their time rates of change, and any single pressure.

For the parametric approach, we are interested in how the payload function depends on the unknown parameters. These unknown parameters are the masses, m_i, and the moments $m_i r_i$ and I_i of the links. Since the energies and generalized forces are linear in these parameters, the payload mass is a rational function of the parameters, linear in both the numerator and the denominator. Hence, the parametric approach entails a non-linear regression analysis.

6.4 PIMS Digger

To compare the parametric and non-parametric approaches, consider the simple digger shown schematically in Figure 6.5. The base is fixed, and the configuration changes only with the angle θ. As there is only a single link of length l which does not rotate, the subscripts are dropped. The centre of mass of the link is assumed to lie on the link at a distance x from the base. However, in addition to the mass of the link, a mass M representing the payload is attached to the end of the link. Thus, $v_1 = 0$, $\omega_1 = \dot{\theta}$ and $r_1 = (x \cos\theta, x \sin\theta)$, and Equations (6.2), (6.3) and (6.4) become

$$T = \frac{1}{2} \dot{\theta}^2 I + \frac{1}{2} M l^2 \dot{\theta}^2 \tag{6.5}$$

$$V = (Ml + mx) \sin\theta \tag{6.6}$$

$$Q = h'(\theta) p A \tag{6.7}$$

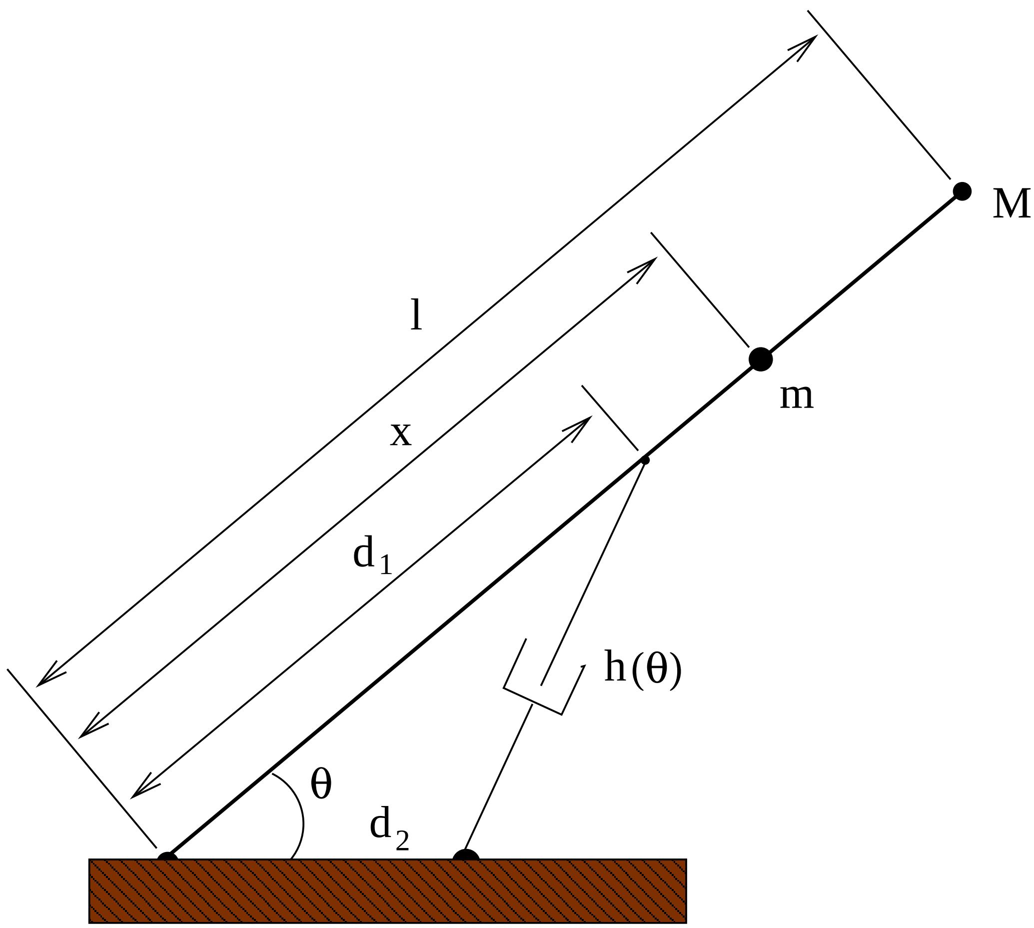


Figure 6.5: The PiMS Digger: a simple, single degree of freedom digger.

Substituting the required derivatives into Equation 6.1, leads to the relation

$$\left(I + Ml^2\right)\ddot{\theta} + g\left(Ml + mx\right)\cos\theta = h'(\theta)pA,$$

which can be solved for the payload mass. This is expressed as

$$M(p,\theta) = \frac{h'(\theta)pA - gmx\cos\theta - I\ddot{\theta}}{l^2\ddot{\theta} + gl\cos\theta}. \tag{6.8}$$

The length h is given by the expression

$$h(\theta) = \sqrt{d_1^2 + d_2^2 - 2d_1d_2\cos\theta},$$

and so

$$h'(\theta) = d_1d_2\sin(\theta)/h(\theta).$$

The lengths d_1 and d_2, which describe the configuration of the piston, are assumed to be known to any desired accuracy. Whereas, c, m I, and l can only be estimated. This is representative of the real case where the centre of mass of the payload cannot be determined in advance simply because the distribution of the load in the bucket is not known. Further, the centre of mass and the inertial tensors of each link cannot be determined for each machine due to economic constraints.

For the dynamic equation, the mass is a non-linear function of the three parameters. To avoid the complications of non-linear regression consider the static case. In this case, the mass can be represented as a function of the pressure, angle and the two unknown parameters, c_1 and c_2 as and Equation 6.8 becomes

$$M(p,\theta) = c_1p\tan\theta/h(\theta) - c_2. \tag{6.9}$$

where $c_1 = d_1d_2A/gl$ and $c_2 = mx/l$. Note, that since many of the unknown parameters cannot be determined to any desired accuracy, non-dimensionalization is not practical. For simplicity, the units kilograms, meters and seconds will be used, as these are relevant to the scale of the mining excavators. Figure 6.6 shows the contours of constant mass for the simple model as a function of the single angle θ and the pressure, p in the single cylinder using the parameter values $c_1 = 10$ m^{2}s^2 , $c_2 = 2$ kg, $d_1 = 2$ m, $d_2 = 1$ m.

In this simple model we have the option of using radial basis function to approximate this function, or using linear regression with *training* data for M, θ and p to estimate the parameters.

To illustrate the effectiveness of the parametric and non-parametric approaches to the problem, two data sets were generated for testing. The first data set was generated by choosing 1000 random points in the p-θ plane, computing the mass, $M(p,\theta)$ from Equation 6.9 and adding a small amount of noise to all three variables. This data is shown in Figure 6.7a. The 'plus' signs represent data used for training the model and the circles represent data used to test the resulting payload function. A second set of data, shown in Figure 6.7b, was generated using only three values of the mass M (20 kg, 30 kg and 40 kg). Equation 6.9 was used to determine the pressure p as a function of the mass M and angle θ. This function was then used to determine the pressures corresponding to the three masses and a random sample of angles. Once again, noise was added to the resulting points to represent real errors in measurement. The data points corresponding to the largest and smallest masses were used for training, and the data for the middle mass was used for verification.

The function $M(p,\theta)$ obtained using approximation by a radial basis functions trained on these two data sets is shown in Figure 6.8. Comparing the two contour plots to the exact contour plot of Figure 6.6 clearly shows the need to train the approximation on a large, uniformly distributed sampling of data.

Since the model is linear in the two parameters, it is straightforward to perform a linear regression using Equation 6.9 to estimate both c_1 and c_2 for each data set. Figure 6.9 shows the contour plots of the approximate functions using the two data sets. Although the errors are much larger for the second set, the parametric method produces far better results than the non-parametric method.

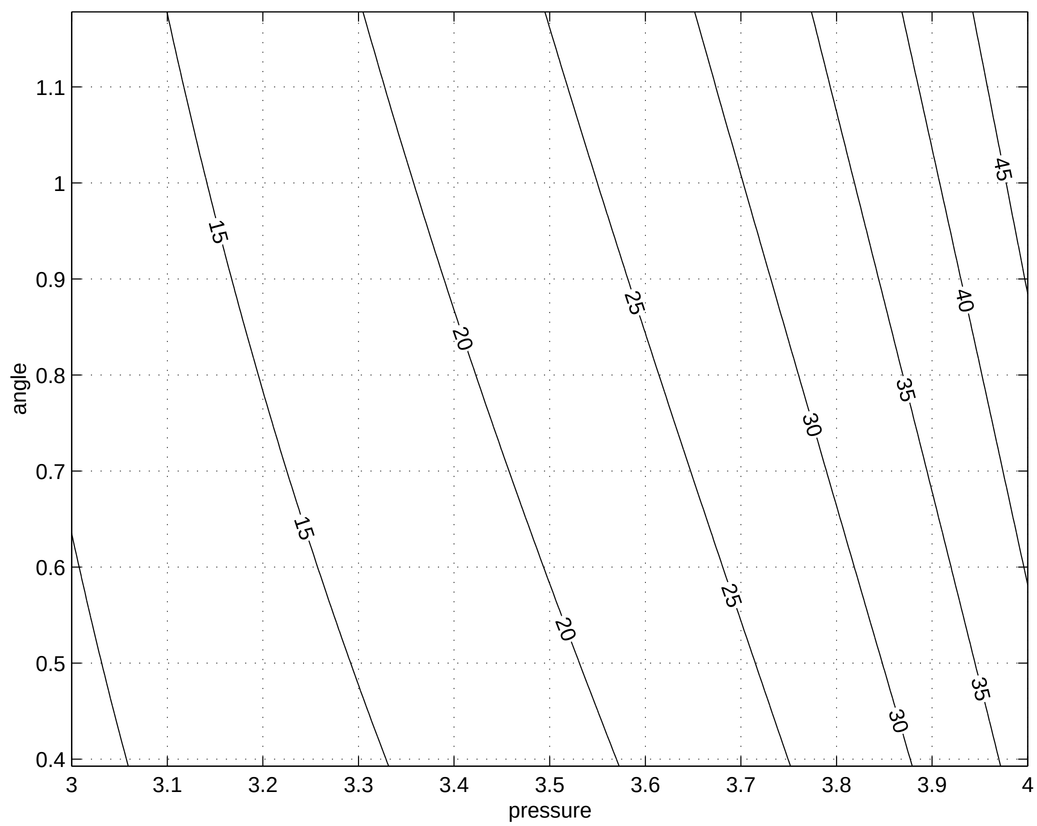
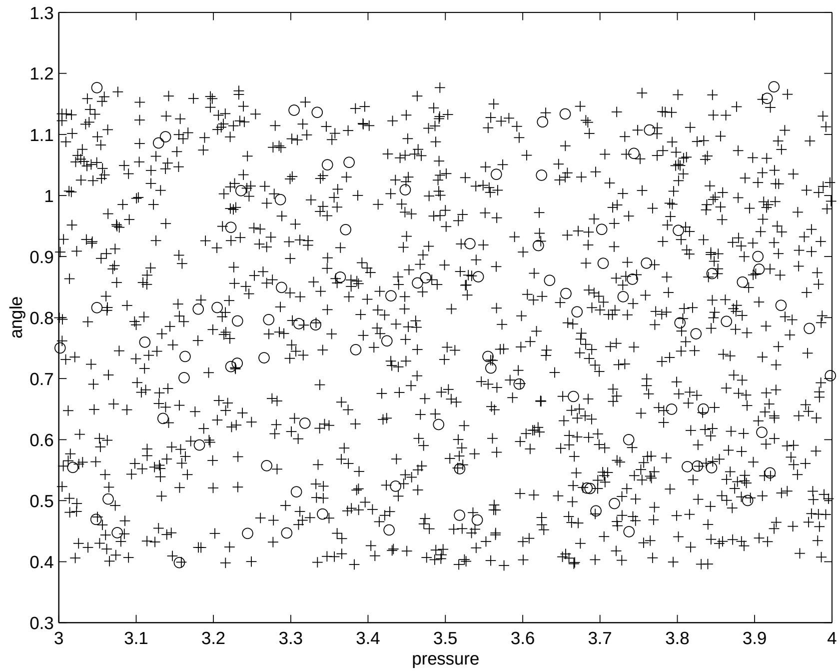
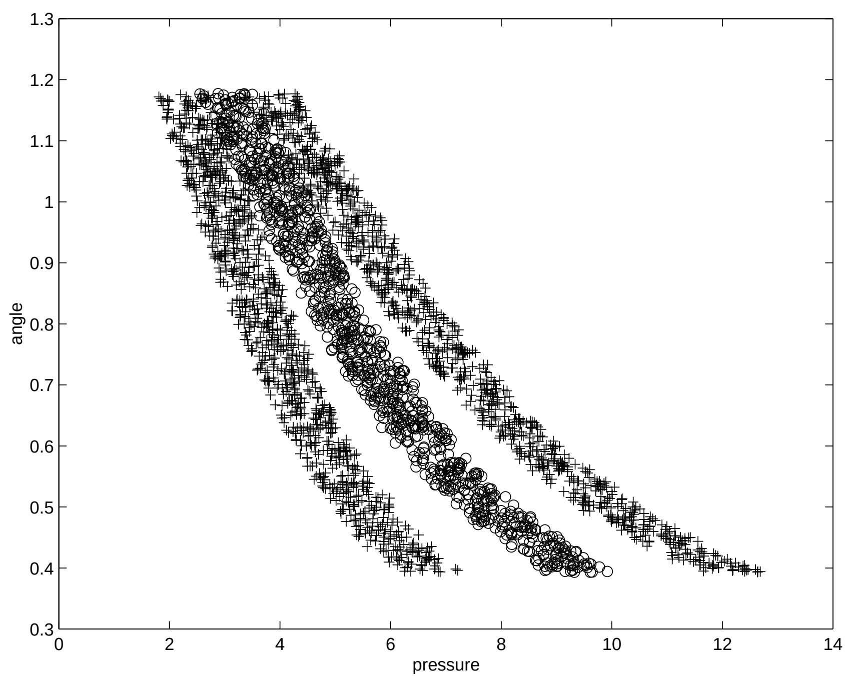


Figure 6.6: Contours of the exact payload function for the PiMS Digger

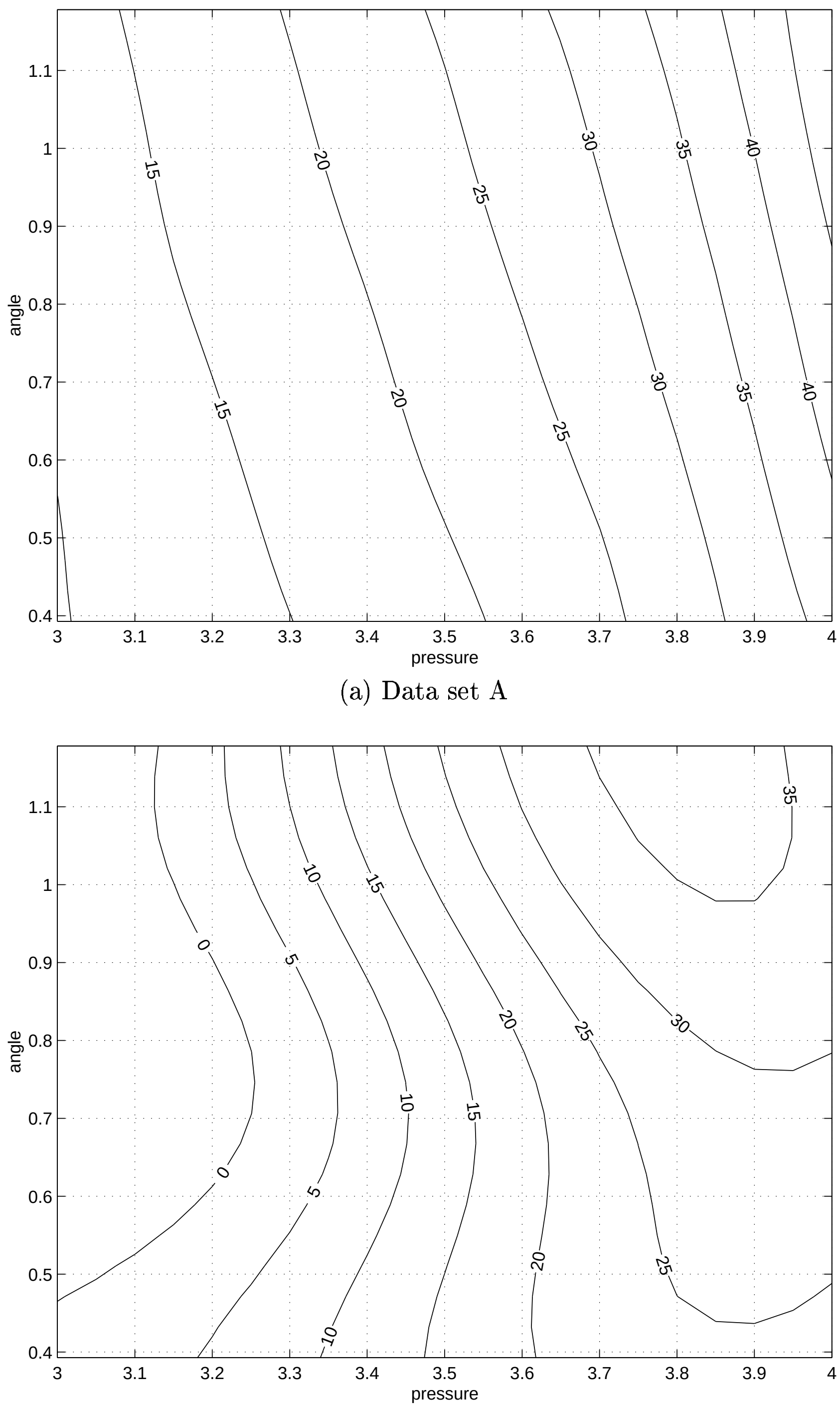


(a) Data set A



(b) Data set B

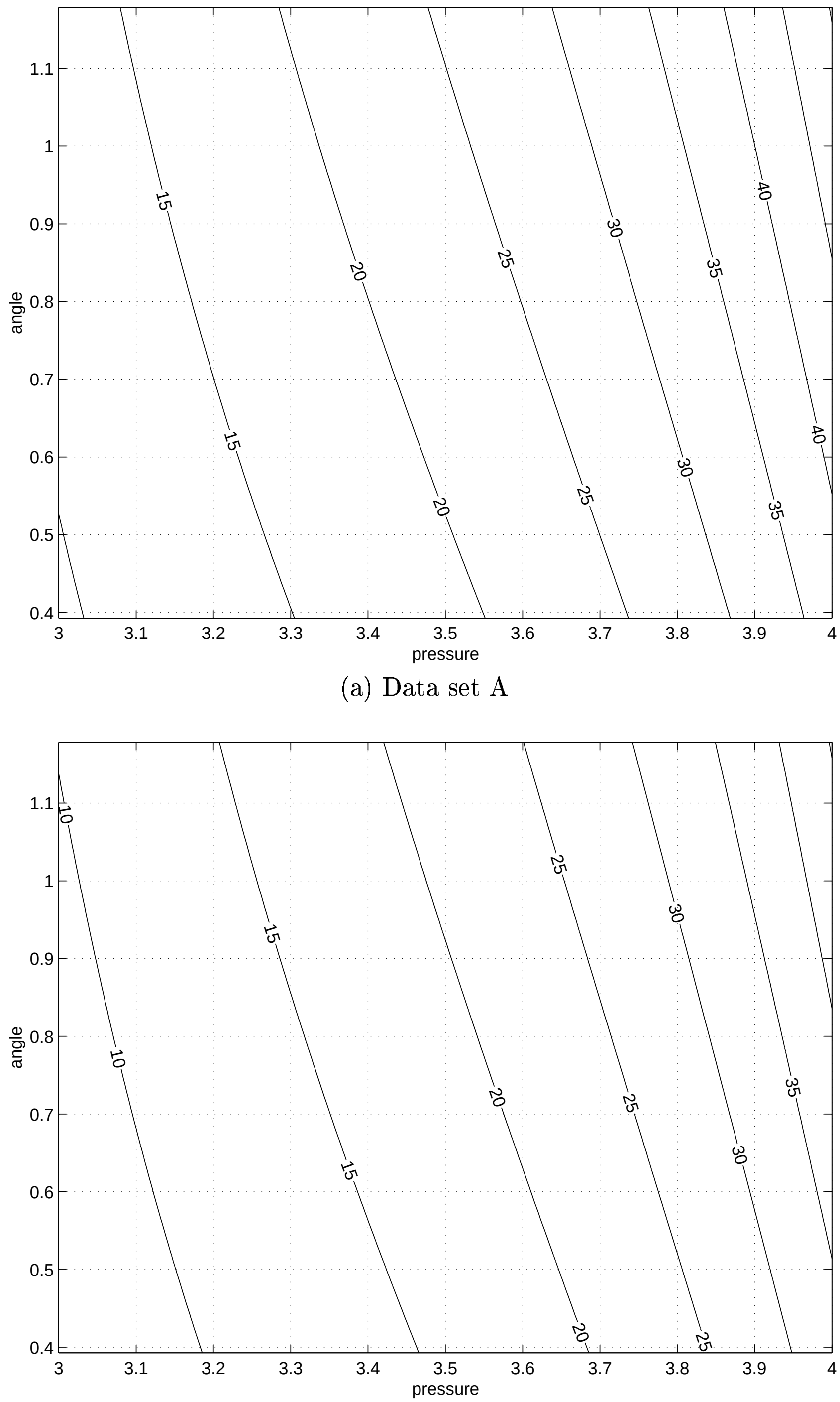
Figure 6.7: Data sets for training and testing of the models. The training data is marked by cross-hairs and the testing data by circles.



(a) Data set A

(b) Data set B

Figure 6.8: Contours of the non-parametric approximations to the payload function for the PiMS Digger. Top, data set A, bottom, data set B



(a) Data set A

(b) Data set B

Figure 6.9: Parametric approximations to the payload function for the PiMS Digger. Top, data set A, bottom, data set B.

6.5 Conclusions

Our testing of both parametric and non-parametric methods of reconstructing the mass function from data indicate that the non-parametric *black box* approach to the problem is not likely to be economically feasible. Although in theory the approach is simple to implement and can be applied to a variety of machines with no additional modelling, in practice, the number of data points necessary for the training of each box is too large. On the other hand, it is clear that the parametric approach can be used to estimate the mass function without knowledge of the exact geometry of the machine, so that a single, parametric, 'grey' box can be applied to a wide variety of machines with the same basic geometry.

In this report we considered only a simple linear model to illustrate the advantages of parametric regression over non-parametric regression for this particular application. For the full machine, it is likely that non-linear regression techniques must be applied. It remains to be shown that these methods will converge for the data available for training. Since the normal operation of the excavator is not to be interfered with, the actual device would necessarily use the full dynamic model rather than the simpler static model explored here. Interestingly, there is less uncertainty in the dynamic motion of the digger arm than in the static situation. In the kinetic case, the frictional force in a cylinder is a function of the piston velocity. In contrast, the magnitude of the static frictional force will depend on the history of motion. That is, even in theory, when frictional forces are included, the mass in the bucket can not be determined from pressure and angle data alone. A continuum of configurations are possible for the same mass.

Bibliography

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