Chapter 3

Determining Geological Properties by a Hybrid Seismic–Magnetotelluric Approach

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3.1 Introduction

Of all the geophysical techniques, controlled-source reflection seismology is the preferred method for oil and gas exploration in the Canadian foothills, as it provides the best resolved images of structures at depth and can provide detailed information about subsurface rock properties such as elastic or acoustic impedance contrasts. Unfortunately, good quality seismic data cannot always be acquired over subsurface targets. The overburden can attenuate or scatter the seismic signal, preventing the signal from reaching the target zone and/or from returning to the detectors. The recorded signal might be swamped by noise, occasionally from external sources (such as production equipment), but more typically generated by interaction of the propagating wavefield with complicated near-surface conditions. In other cases, access to the surface over the target area is restricted and the resulting data set provides limited information about the target

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and/or the overburden, both necessary prerequisites for high resolution imaging of impedance contrasts in the subsurface.

The controlled source audio magnetotelluric technique (CSAMT) is another viable geophysical tool for imaging subsurface structure. Unfortunately, electromagnetic waves propagating in conducting media satisfy the diffusion equation, and hence the best possible resolution of subsurface conductivity structure using the MT method is typically much worse than for seismic waves propagating in an elastic media. However, great strides have been made over the last decade in the acquisition, processing, interpretation and inversion of 2D and 3D MT data. It might be possible to collect high quality MT data in areas where the seismic data are of poor quality. In situ well logs that measure rock properties at the target and in the overburden show that the structural boundaries separating zones of different conductivity correspond exactly with major contrasts in elastic impedance. Thus, an inversion of the MT data constrained by a priori information obtained from seismic and well data or a joint inversion of both data sets together could provide a better image of the subsurface structure than the seismic alone.

Shell Canada posed a series of questions for consideration, all variations on a similar theme. These questions can be grouped in order from more general to more specific.

1. Could MT help to better define the geology, in comparison to using a regional geological model, in areas of poor seismic?
2. Can a geological model be produced by joint tomographic inversion of both data types?
3. Which would be favorable conditions for the application?
4. What kind of computing power is likely to be involved?
5. Given the recent advances in the acquisition, processing, interpretation and inversion of MT data and the associated improvement in lateral resolution, is there an emerging opportunity to combine MT with seismic in areas of complex geology?
6. What are the theoretical and practical issues involved in combining surface seismic and surface MT data by joint tomographic inversion in areas where seismic alone cannot sufficiently explain the subsurface in the Canadian Foothills.
7. Is common inversion of seismic and magnetotelluric data an option for gas exploration in the heavy structured thrust belt environment of the Canadian Foothills?

To properly address questions 4–7 would require an expert knowledge of 2D and 3D MT and seismic reflection methods, as well as software for forward modeling and inversion. Our group determined that these resources were not readily available, or if available could not be utilized in the short time span available. Therefore, we decided to focus our efforts on question 1–3. We limited our investigation to a simple 1D earth model where regional seismic and well logs suggest discrete layers, each with constant seismic velocity and constant electrical conductivity. In addition, the well logs provide rough estimates of velocity and conductivity for use as a starting point in the seismic and MT inversions.

To the best of our knowledge, there are no previous works that attempt a joint inversion of both MT and seismic data, although our joint inversion scheme closely follows the generalized methodology proposed by Lines et al. (1988). Ellis (1999) shows that joint inversion of airborne EM data and pole-pole dc resistivity data can effectively reduce non-uniqueness in EM inversion. Sasaki (1989) shows that joint 2D inversion of MT and dipole-dipole resistivity can effectively improve the resolution compared to that from either of the separate inversions. More typically, a-priori information from seismic and well data are introduced as constraints on the lateral roughness of boundaries between abrupt changes in conductivity, and on the smoothness of conductivities within layers (Smith et al. 1999). Newman et al. (2002) have published the most comprehensive investigation to date of seismic-constrained MT inversion. In addressing a complex 3D subsalt imaging problem, they apply constraints that include fixed and floating boundaries as well as fixed values and/or lower bounds for conductivities above and within the salt. Routh and Oldenburg (1999) discuss the specific problem of 1D inversion for CSAMT, while Li et al (2000) examine 1D inversion for CSAMT with azimuthal anistropy.

In this paper, we focus more on the magnetotelluric problem, as it is somewhat less well-known. A deeper understanding of this model will be important to further studies in the feasibility of joint seimic-MT inversions.

3.2 Justification for 1D earth model

Although our 1D earth model is a gross simplification of the complex 2D and 3D geologic structure typical of the Canadian Foothills, an investigation of the joint inversion problem for 1D earth structure is necessary and useful. Most 2D and 3D MT inversion schemes require a reasonable starting model. The usual practice is to create the starting model from a profile of constrained 1D MT inversions. A robust procedure for the 1D inversion problem is unlikely to yield subsurface models suitable for oil and gas exploration, but might yield an intermediate model suitable for 2D and 3D MT inversions. A very similar argument applies to the seismic imaging problem, but it is worthwhile discussing in detail as there are important differences between the two.

Conventional seismic imaging is achieved by migrating the data. Migration is best described as a data processing step that convolves a space-time filter with the seismic data. The image can be created in a single pass, or in a series of passes that iteratively reshape the data towards the desired image. In short, then, the subsurface reflectivity model is obtained by migration, not inversion. However, one of the most important parameters controlling the behaviour of these filters is velocity. Hence, we require a reasonable velocity model of the subsurface, and it is towards this end that our inversion efforts are directed. In fact, one of the attractions of seismic migration is that a well-focused image of the subsurface can often be obtained with a very approximate velocity model (Geiger 2001). Thus, the output from our inversion process – a simple 1D layered velocity model – can be considered as an intermediate product in the quest for more detailed 2D and 3D velocity models, and these in turn as parameters controlling the behaviour of the migration operators.

3.3 Basic seismic wave theory

It is well known (e.g. Wapenaar and Berkhout, 1989) that acoustic and elastic wavefields satisfy the scalar wave equation. Although earth materials are elastic, most industry applications of the seismic reflection method only consider propagation of compressional waves, whose behaviour is adequately described by the acoustic wave equation for pressure. In an isotropic (i.e. material properties do not depend on direction of wave-propagation), source-free region, pressure P satisfies the homogeneous wave equation in the frequency domain:

$$(\nabla^2 + k^2)P = 0 \tag{3.1}$$

where $k = \omega\sqrt{\rho/K}$ is the wave number for frequency ω, density ρ , and adiabatic compression modulus K. Equation (3.1) is a scalar Helmholtz equation that describes the propagation of acoustic waves in real earth materials.

For a plane wave propagating along the z axis (positive down), equation (3.1) has solution

$$P = P^+e^{ikz} + P^-e^{-ikz}. \tag{3.2}$$

If we express the time dependence of the harmonic wave for positive frequencies explicitly as $e^{-i\omega t}$, i.e.

$$P = P^+e^{i(kz-\omega t)} + P^-e^{-i(kz-\omega t)}, \tag{3.3}$$

we see that the first and second terms on the RHS of equations (3.2) and (3.3) represent downgoing waves (for z positive) and upgoing waves (for z negative), respectively. The phase velocity $\omega/k = \sqrt{K/\rho} \equiv \nu$ is only a function of the material properties and is independent of frequency, thus the acoustic wave propagation is non-dispersive. Note as well that, in a homogeneous media, propagating acoustic plane waves do not suffer attenuation. These properties are the fundamental reason for the superior resolution of the seismic methods.

3.4 Forward modelling of seismic reflection data

In essence, reflection seismology is a remote-sensing technique based on the principles of echolocation. An energy source at the surface of the Earth introduces sound waves that propagate through the various rock layers in the subsurface. Abrupt changes in the density and adiabatic compression moduli – which typically correspond to layer boundaries or structural discontinuities – impede the transmission of the sound waves. At each impedence constrast, or ‘reflector, a small fraction of the propagating energy is reflected or diffracted and returns to the surface as a faint echo of the source. Receivers that measure acoustic pressure or particle motion detect these ‘reflection events. The corresponding amplitudes are digitally recorded as time series known as ‘seismograms. Because the signal propagates without significant dispersion or attenuation (other than geometrical spreading effects), an impulsive source will generate seismograms with well defined reflection events. The seismograms can be processed to create an image or representation of the subsurface.

A high-frequency approximation to the forward modeling operator (known as ray theory) can accurately describe wave propagation as a linear superposition of traveltime-delayed and amplitude-scaled copies of the source signal, one for each raypath linking a given source with a subsurface reflector element and back to a receiver. Because the experiment is bandlimited, the infinitely many raypaths to all possible subsurface reflectors can be approximated by a finite subset (by discrete sampling of each subsurface reflectors). This subset of raypaths can be further reduced by the application of stationary phase, with the result that the vast majority of the recorded signal comes from specular reflections where the angles of incidence and reflection at a given reflector element are equal. For simple earth models such as the 1D layer case, the forward modeling problem for one source and receiver reduces to calculation of a single traveltime along a piecewise straight raypath. In most applications, the amplitude correction is ignored, or approximated by an estimate of the geometrical spreading along the raypath. We adopt this methodology to forward model the seismic data.

3.5 Inversion of seismic data

The fundamental inverse problem for seismic reflection consists of estimating acoustic or elastic impedance contrasts (angle dependent reflectivity) as a function of position (Gray, 1997 and Bleistein et al, 2001). However, as discussed above, the image is conventionally produced by migration (data processing) rather than via a true mathematical inversion. Our interest is in obtaining a velocity model that represents useful parameters for the migration model. In our model here, we examine the seismic data and pick travel times, which represent the arrival times for give seismic signals returning after relection from geological layers. These travel times are then used as inputs to the inversion problem for the velocity model.

3.6 Basic electromagnetic wave theory

Consider Maxwells equations in the frequency domain for an isotropic, source-free region with the constitutive relations $\vec{D} = \varepsilon\vec{E}$, $\vec{B} = \mu\vec{H}$, and $\vec{J} = \sigma\vec{E}$:

$$\vec{\nabla} \times \vec{H} = (\sigma - i\omega\varepsilon)\vec{E} \tag{3.4}$$

$$\vec{\nabla} \times \vec{E} = i\omega\mu\vec{H} \tag{3.5}$$

$$\vec{\nabla} \cdot \vec{H} = 0 \tag{3.6}$$

$$\vec{\nabla} \cdot \vec{E} = 0 \tag{3.7}$$

for frequency ω, magnetic permeability μ, dielectric permittivity ε, and electrical conductivity σ. The inverse of conductivity is known as electrical resistivity $\rho = \sigma^{-1}$, which should not be confused with density above.

In a homogeneous, isotropic, source-free region, both the electric field $\vec{E}$ and magnetic field $\vec{H}$ can be shown (e.g. Ward and Hohmann, 1988) to satisfy the homogeneous wave equation in the frequency domain

$$(\nabla^2 + k^2)\vec{E} = 0 \tag{3.8}$$

$$(\nabla^2 + k^2)\vec{H} = 0 \tag{3.9}$$

where k is the propagation constant or wave number, defined as $k^2 = i\omega\mu\sigma$ at low frequencies where conduction currents predominate over displacement currents. Equations (3.8) and (3.9) are vector Helmholtz

For a plane wave propagating along the z-axis (positive down), equations (3.8) and (3.9) have solutions

$$\vec{E} = \vec{E}^{+}e^{ikz} + \vec{E}^{-}e^{-ikz}, \tag{3.10}$$

$$\vec{H} = \vec{H}^{+}e^{ikz} + \vec{H}^{-}e^{-ikz}. \tag{3.11}$$

Expanding $k = \alpha + i\alpha$, where α is the positive real quantity $\sqrt{\omega\mu\sigma/2}$, and expressing the time dependence of the harmonic wave for positive frequencies explicitly as $e^{-i\omega t}$, we see that the first terms on the RHS of equations (3.10) and (3.11) represent downgoing waves (for z positive)

$$\vec{E}^{+}e^{ikz}e^{-i\omega t} = (\vec{E}^{+}e^{-\alpha z})e^{i(\alpha z-\omega t)}, \tag{3.12}$$

while the second terms on the RHS of equations (3.10) and (3.11) represent upgoing waves (for z negative)

$$\vec{E}^{-}e^{-ikz}e^{-i\omega t} = (\vec{E}^{-}e^{\alpha z})e^{-i(\alpha z+\omega t}, \tag{3.13}$$

In both cases, the electromagnetic waves attenuate as they propagate. In addition, the phase velocity $\omega/\alpha = \sqrt{2\omega/\mu\sigma}$ is frequency dependent, even in common earth materials where conductivity and permeability are independent of frequency. Hence, electromagnetic waves propagating in materials with non-zero conductivity are dispersive. Dispersion and attenuation are the reasons for the lack of resolution of electromagnetic prospecting methods.

3.7 Forward modelling of magnetotelluric data

From the vanishing of the divergences (equations (3.6) and (3.7)), the fields lie completely in the x-y plane, so that

$$\vec{E} = (E_x, E_y, 0), \tag{3.14}$$

$$\vec{H} = (H_x, H_y, 0). \tag{3.15}$$

From the curl equation (3.5), the components of the two fields are related by

$$H_x(z) = -\frac{k_n}{\omega\mu}(E_y^{+n}e^{ik_nz} - E_y^{-n}e^{-ik_nz}) \tag{3.16}$$

$$H_y(z) = \frac{k_n}{\omega\mu}(E_x^{+n}e^{ik_nz} - E_x^{-n}e^{-ik_nz}) \tag{3.17}$$

where the subscript n indicates a specific layer in a layered media with conductivities σ_n. Equations (3.16) and (3.17) are only valid within the limits of the layer with index n.

In the Tikhonov-Cagnaird approach, the wave impedance is defined as

$$Z_{xy}(z) = \frac{E_x(z)}{H_y(z)} \tag{3.18}$$

$$Z_{yx}(z) = \frac{E_y(z)}{H_x(z)} = -Z_{xy}(z) \tag{3.19}$$

i.e. the ratio of orthogonal components of the electric and magnetic fields. Note that the wave impedance will be a complex quantity, and thus have both amplitude and phase.

For a homogeneous half-space, there are no upgoing waves. Substituting the downgoing x-component of equation (3.10) and the downgoing portion of equation (3.17) into equation (3.18) yields

$$Z_{xy} = \sqrt{\frac{-i\omega\mu}{\sigma}}. \tag{3.20}$$

Similarly, substituting the downgoing y-component of equation (3.11) and the downgoing portion of equation (3.16) into equation (3.19) yields

$$Z_{yx} = -\sqrt{\frac{-i\omega\mu}{\sigma}}. \tag{3.21}$$

For normally conducting rocks $\mu = \mu_0$, thus the observed impedance is dependent only on frequency and the electrical conductivity in the earth. It is not dependent either on the strengths of the individual components of the electromagnetic field or the measurement depth z. Thus measurements taken at the surface of the earth ($z = 0^+$) are sufficient to determine conductivity (or its inverse, resistivity). The resistivity of the half-space can be determined from the field by

$$\rho = \frac{1}{\sigma} = \frac{1}{\omega\mu}\frac{|E_x|^2}{|H_y|^2} = \frac{1}{\omega\mu}\frac{|E_y|^2}{|H_x|^2}. \tag{3.22}$$

For layered media, Tikhonov and Cagniard both showed (see e.g. Zhdanov and Keller, 1994, p262-269) that the impedance measured at the surface of the earth can be expressed as

$$Z_{1D} = \sqrt{\frac{-i\omega\mu}{\sigma_1}}C_N, \tag{3.23}$$

where C_N is the layered earth correction factor for the plane wave impedance for the layered earth as a whole. C_N is a function of the layer thicknesses and propagation constants in the subsurface, found by matching continuity conditions at each layer interface $z = h_n$ (both H_x and E_y, or H_y and E_x are continuous acrross layer interfaces).

Cagniard defined an apparent resistivity ρ_a in terms of the observed wave impedance at the earth's surface as

$$\rho_a = \frac{1}{\omega\mu}|Z|^2, \tag{3.24}$$

a quantity often used instead of the wave impedance Z_{1D}.

For the 1D magnetotelluric method, the ideal data set consists of measurements of the orthogonal horizontal components of the electric and magnetic field over a range of frequencies, and occasionally the vertical magnetic field (see discussion of tipper vector below). This allows determination of both amplitude and phase of the impedance. The phase of the impedance, although not been discussed above, is a valuable measurement because it can help constrain 1D inversions in the presence of noise and static shifts. However, it is often sufficient to have measurements of the apparent resisitivity over a range of frequencies, as we will assume here.

3.8 Inversion of magnetotelluric data

The fundamental inverse problem for MT consists of estimating conductivity as a function of position (Hohmann and Raiche, 1988; Whittall and Oldenburg, 1991). MT is considered a classical geophysical inverse problem. The MT inverse problem is non-linear and ill-posed. For any real subsurface distribution of conductivity and finite data set, the problem is underdetermined (more model parameters than data). In order that a solution of any sort can be achieved, the subsurface is simplified so that the problem is overdetermined (more data than model parameters). Thus there are a number of possible solutions, all of which might fit the data equally well. An additional contribution to non-uniqueness is the presence of random noise in the data, although repeatable geologic noise can manifest itself as static shifts (bias) in measured field components, resulting from near surface inhomogeneities (Wannamaker, 1999; Garcia and Jones, 2002), as well as fine-scale variations in resistivity-depth profiles as seen in well logs (White et al., 2001).

All MT inversions are constrained in some sense. The constraints might include careful preprocessing and/or the selection of subsets from the available data (Agarwal et al., 1993; Oldenburg and Ellis, 1993; Berdichevsky et al., 1998; Pedersen and Gharabi, 2000), choice of a particular class of models (e.g. smoothest model: Constable et al., 1987; Smith and Booker, 1988, or minimum blocky model: Fischer and Weibel, 1991), selection of a means for searching the model space to find the best fit model (Weaver and Agarwal, 1993; Grandis et al. 1999, Siripunvaraporn and Egbert, 2000), and the introduction of a priori bounds that limit or guide the fitting process (Whittall, 1986; Mackie and Madden, 1993; Smith et al. 1999; Newman et al. 2002) .

3.9 Practical considerations for 1D MT

In most cases, the earth cannot be adequately described by a 1D model. However, reasonable 1D inversions might still be desirable, if only to provide a good starting model for further 2D and 3D inversions. In addition, even in areas where a 1D model might be appropriate, near surface inhomogeneities distort the measured electric and magnetic fields and bias the calculation of impedances and apparent resistivities.

In areas of lateral inhomogeneity and/or anisotropy, the electric field in one direction may depend on magnetic field variations parallel to, as well as perpendicular to, its direction. The impedance is a tensor (see, e.g. Vozoff, 1991), given by

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix} \tag{3.25}$$

or, equivalently,

$$\mathbf{E} = \mathbf{Z}\mathbf{H}. \tag{3.26}$$

In a 2D subsurface where geologic strike is parallel to the x-axis and perpendicular to the y-axis, the diagonal elements Z_{xx} and Z_{yy} are zero, and $Z_{xy} \neq -Z_{yx}$. Impedance Z_{xy} corresponds with an incident electric field along x, which is often referred to as the transverse electric (TE) mode. The E_x field of the TE mode induces no boundary or space charges and so there are no boundary charge effects. Impedance Z_{yx} corresponds with an incident electric field along y, which is often referred to as the transverse magnetic mode or TM mode. The E_y field of the TM mode crosses boundaries, so that charges are induced, and anomalies persist to arbitrarily long periods. Thus the TM impedance is more sensitive to near-surface resistive structures than the TE impedance. Agarwal et al. (1993) examine the two common effective impedances which are rotationally invariant in the x-y plane:

$$Z_{ave} = \frac{1}{2}(Z_{xy} - Z_{yx}), \tag{3.27}$$

$$Z_{eff} = (Z_{xx}Z_{yy} - Z_{xy}Z_{yx})^{1/2}. \tag{3.28}$$

These are valid for any orientation of the acquisition geometry relative to geologic strike, although they conclude that the TE impedance is more suitable for 1D inversions in 2D media (e.g. to generate a starting model for 2D inversions).

Near lateral conductivity changes, $\vec{\nabla} \times \vec{E}$ has a vertical component, thus H_z is not zero and the $\vec{H}$ vector is tipped out of the horizontal plane. A vertical magnetic transfer function or tipper (A, B) can be defined as

$$H_z = AH_x + BH_y. \tag{3.29}$$

The tipper has both amplitude and phase and is frequency dependent. Given a 2D structure with strike, the tipper can help resolve ambiguity in strike and show which side of a contact is more conductive (Vozoff, 1991).

In general, near-surface inhomogeneities introduce galvanic distortion effects that can be described by distortion matrices that relate the regional electromagnetic fields to the measured (superscript D) distorted fields (see Garcia and Jones, 2002; Li et al., 2000):

$$\mathbf{E}^D = \mathbf{PE} \tag{3.30}$$

$$\mathbf{H}^D = \mathbf{Q}_h\mathbf{E} = \mathbf{I} + \mathbf{Q}_h\mathbf{Z} \tag{3.31}$$

$$H_z^D = [(A, B) + \mathbf{Q}_z\mathbf{Z}]\mathbf{H} \tag{3.32}$$

where $\mathbf{I}$ is the identity matrix, and $\mathbf{P}$, $\mathbf{Q}_h$ and $\mathbf{Q}_z$ are real valued and frequency-independent distortion matrices:

$$\mathbf{P} = \begin{pmatrix} P_{xx} & P_{xy} \\ P_{yx} & P_{zz} \end{pmatrix}, \mathbf{Q}_h = \begin{pmatrix} Q_{xx} & Q_{xy} \\ Q_{yx} & Q_{zz} \end{pmatrix}, \text{ and } \mathbf{Q}_z = \begin{pmatrix} Q_{zx} & Q_{zy} \end{pmatrix}. \tag{3.33}$$

When the EM fields are measured in an arbitrary coordinate system relative to the geologic strike, the observed impedance $\mathbf{Z}^D$ is related to the desired regional 2D impedance $\mathbf{Z}_{2D}$ by

$$\mathbf{Z}^D = \mathbf{R}(\theta)(\mathbf{I} + \mathbf{P})\mathbf{Z_{2D}}(\mathbf{I} + \mathbf{QZ})^{\mathbf{-1}}\mathbf{R^T}(\theta), \tag{3.34}$$

where $\mathbf{R}(\theta)$ is the Cartesian rotation tensor, and T denotes transpose. Theoretical and practical aspects of the decomposition are discussed in McNeice and Jones (2001), Garcia and Jones (2002), and Wannamaker (1999) (as well as references therein).

3.10 Workshop Model

At the IPSWorkshop, we choose to investigate a simple one-dimensional model, to get some idea of how the the joint seismic/magnetotelluric inversion might proceed. We consider a hybrid seismic-magnetotelluric method for determining geological properties using a one-dimensional three-layer model. We assume that the medium under consideration has three horizontal layers, each of them is homogeneous with constant layer thickness h_n, resistivity ρ_n and seismic velocity v_n, $n = 1, 2, 3$ from top down, and layer three has infinite thickness, i.e., $h_3 = \infty$.

3.10.1 Seismic method

Forward problem. Seismic signals are produced at certain location on the surface of the top layer ($z = 0$) and reflections of the signals from the interfaces of layers ($z = h_1$, h_2) are recorded at different locations. Suppose that the distance between the source and the jth receiver is $2x_j$, then $T_{n,j}$, the time for the signal to reach the receiver j, after nth reflection, can be computed as

$$T_{1,j} = \frac{\sqrt{x_j^2 + h_1^2}}{v_1}, \tag{3.35}$$

$$T_{2,j} = \sqrt{2\left(\frac{h_1}{v_1} + \frac{h_2 - h_1}{v_2}\right)\left(1 + \frac{2x_j^2}{h_1 v_1 + (h_2 - h_1) v_2}\right)} \tag{3.36}$$

where v_n are the seismic velocities in layer n.

Inverse problem. In practice, the situation is the opposite. We can record the travel time $T_{n,j}$ at location j and often the layer thickness and seismic velocity are not known. The question is how to find h_n and v_n based on the values of $T_{n,j}$.

The solution of the inverse problem is typically non-unique and sometimes unstable. The main challenge is to find the correct solution or an approximation efficiently.

3.10.2 Magnetotelluric method

Forward problem. Assume that the electric-magnetic field satisfies the Maxwell equations and the traveling wave solution satisfies

$$\nabla \times \vec{E} + i\mu\omega\vec{H} = 0, \quad \nabla \times \vec{H} - \sigma\vec{E} = 0 \tag{3.37}$$

where ω is the frequency of the wave, $\sigma = \rho^{-1}$ is the conductivity and μ is the permeability of the medium. The equations are equivalent to the Helmholtz equations for $\vec{E}$ and $\vec{H}$ when there are no sources/sinks inside the domain (i.e., both $\vec{E}$ and $\vec{H}$ are divergence-free) and in our one-dimensional setting they are

$$\frac{d^2 E_n}{dz^2} + k_n^2 E_n = 0, \tag{3.38}$$

$$\frac{d^2 H_n}{dz^2} + k_n^2 H_n = 0 \tag{3.39}$$

where E_n and H_n are the x and y components of the vector fields $\vec{E}_n$ and $\vec{H}_n$ in layer n, respectively and $k_n^2 = -i\sigma_n\mu\omega$.

The solutions for E_n and H_n can be obtained as

$$E_n = A_n \exp(ik_n z) + B_n \exp(-ik_n z), \tag{3.40}$$

$$H_n = \frac{k_n}{\omega\mu}\left(A_n \exp(ik_n z) - B_n \exp(-ik_n z)\right). \tag{3.41}$$

When H_1 (or E_1) is given, the coefficients A_n and B_n can be determined by matching conditions at each layer interface $z = h_n$ (both H_n and E_n are continuous across the layer interfaces). Note that $B_3 = 0$ since otherwise the solution will grow unbounded as $z \to \infty$. And each solution E_n and H_n can be expressed by H_1 (or E_1), the layer thickness h_n, the conductivities σ_n, and the frequency of the wave. In particular, the ratio of the electric and magnetic field strengths at the surface $(z - 0)$ are uniquely determined by h_n and σ_n as

$$Z \equiv \left.\frac{E_1}{H_1}\right|_{z=0} = \frac{\omega\mu}{k_1}\frac{1 - \sqrt{\frac{\sigma_2}{\sigma_1}}R\tanh(ik_1h_1)}{\sqrt{\frac{\sigma_2}{\sigma_1}}R - \tanh(ik_1h_1)} \tag{3.42}$$

where

$$R = \frac{1 - K_{23}\exp(i2k_2H_{21})}{1 + K_{23}\exp(i2k_3H_{21})}, \quad K_{23} = \frac{k_2 - k_3}{k_2 + k_3}, \quad H_{21} = h_2 - h_1.$$

A more conventional quantity called the *apparent resistivity*

$$\rho_{app} = \left|\frac{i}{\omega\mu}Z^2\right| \tag{3.43}$$

is normally used instead of the ratio Z.

Inverse problem. Again, often we can measure the apparent resistivity ρ_{app} corresponding to various frequencies. To obtain the geological information such as the layer thickness h_n and resistivity $\rho_n = \sigma_n^{-1}$ based on ρ_{app} is of more practical importance.

3.10.3 A Hybrid Seismic-Magnetotelluric Approach

In practice, there are situations when one set of data (seismic or magnetotelluric) is more difficult to obtain than others. It will be of interest to explore the feasibility of a hybrid approach which combines both data.

We consider the following approach for the inversion of seismic and magnetotelluric data by minimizing the objective function

$$J = \sum_{n,j} w_{n,j}^1 \left(T_{n,j}^m - T_{n,j}^c\right)^2 + \sum_j w_{n,j}^2 \left(\rho_{app}^m(\omega_j) - \rho_{app}^c(\omega_j)\right)^2 \tag{3.44}$$

where $T_{n,j}^m$ and ρ_{app}^m are measured traveling time for the seismic signal and apparent resistivity, $T_{n,j}^c$ and ρ_{app}^c are the computed values using (3.35), (3.36) and (3.43).

In the following, we present numerical experiments which illustrate the importance of the initial guess and the weight functions w^1 and w^2. In general the results suggest that we should always over-estimate the layer thickness for the initial guess. And it is necessary to use a staged procedure in order to find the correct solutions.

3.11 Numerical Experiments

To test the hybrid approach, we replace the measured data $T^m_{n,j}$ and ρ^m_{app} by the exact solutions obtained using (3.35), (3.36) and (3.43) based on the following parameter values: $h_1 = 400$, $h_2 = 4000$, $v_1 = 6332$, $v_2 = 4114$, $\rho_1 = 8000$, $\rho_2 = 40$, $\rho_3 = 8000$ and $\mu = 10^{-7}$. We assume that the seismic data is recorded at nine locations with distance between the source and receiver varying between 1000 and 5000 with equal increments:

$$2x_j = 1000 + \frac{j-1}{8}4000, \quad j = 1, ...9.$$

The magnetotelluric data is recorded at the surface ($z = 0$) for 25 frequencies varying between 0.001 and 500 exponentially

$$\omega_j = 0.001\left(\frac{500}{0.001}\right)^{\frac{j-1}{24}}, \quad j = 1, ...25.$$

The minimization is carried out using the matlab routine `fminsearch` with maximum iteration number set at 2000, maximum number of function evaluations at 10000, convergence criteria for the function value at 10^{-8}. Table 3.1 listed the results for various computations with initial guess $h_1 = h_2 = 10000$, $v_1 = iv_2 = 10000$, $\rho_1 = 6000$, $\rho_2 = 600$, and $\rho_3 = 6000$. We have chosen equal weights for both the seismic and magnetotelluric data in the functional J in (3.44) with two parameters w^1 and w^2. For two of the computations ($w^2 = 100$ and 1000), the program is re-started using the results from the previous calculation as initial guess since some of the corrected values are obtained after the first attempt. It can be seen that the hybrid approach is effective when appropriate weights are chosen. Even though the first iteration does not yield all the correct solution, a second attempt does recover the rest of the solution for the case of $w^2 = 100$. However, for $w^2 = 1000$, the iteration diverges (second row from the bottom) and it converges only after taking the absolution value of the results from the first attempt as an initial guess (first row from the bottom).

3.12 Conclusions

Both seismic imaging and magnetotellurics are well-developed technologies with a rich research literature and history of development; in our one-week exercise, we could only expect to touch on some basic ideas of the methods and make some initial attempts to understand both methods and link them in a hybrid approach. With numerical experiments, we were able to demonstrate a basic understanding of both forms of inversion for a 1D earth model, and make some investigations on joint inversion using a weighed least squares technique. Even this simple method demonstrates promise for combining both seismic and MT data to give an effective inversion method. Weighting for the least squares is significant for balancing the contribution of data from the two methods. A more realistic experiment would involve real data from a 2D seismic and MT survey, but using the same methodology of least squares.

It is worth noting that an extensive literature on seismic and MT imaging methods exists, and a familiarity with this work is critical to making further progress on joint inversion. We include a representative sample of some key research articles in the bibliography below.

Table 3.1: *Effect of the weights w^1 and w^2 on the solution of the inverse problem using the hybrid approach. For comparison purpose, the seismic only ($w^2 = 0$) and magnetotelluric only ($w^1 = 0$) inversion are also included.*

w^1	w^2	h_1	h_2	v_1	v_2	ρ_1	ρ_2	ρ_3
1	0	-400	4061	6332	4455	-	-	-
0	100	0	3738	-	-	0	39	8745
1	10	-398	4062	6330	4455	8001	51	20387
1	100	400	4000	6332	4114	322	38	20285
		400	4000	6332	4114	8000	40	8000
1	1000	397.7	3998.6	6330	4114.1	-4256.3	40.1	7523.5
		399.2	3999.5	6331.3	4114	-2.143×10^{12}	40	7828.4
		400	4000	6332	4114	8000	40	8000

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