Chapter 3

The Effects of Impact on Design Features

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3.1 Introduction

The system of interest is an injection molding machine consisting of a hydraulic piston that forces some molten metal into a mold; see Figure 3.1. The injection piston itself consists of a flange (the tail end), a piston rod, and a screw that is attached to the tip of the piston rod. A variable hydraulic force, that is applied to the piston, attempts to keep the piston moving at a constant velocity, where a maximum force of $F_{\max}$ can be applied. A molten metal (e.g., a magnesium alloy) is injected into a mold, which is pictured at the right side of Figure 3.1. In normal operation the piston screw, travelling at the prescribed velocity, will impact the molten metal that has been injected into the mold and force the molten metal to completely fill the mold. The whole machine is symmetric under rotation about the lengthwise axis of the piston.

It is desired that the machine be designed for (essentially) infinite life. Therefore, design features must be specified so that the machine can withstand the repeated strain on the piston due to the impact of the piston screw on the molten metal. In addition, in the event that there is an insufficient amount of material in the mold, the piston may "bottom out". That is, the flange of the piston may impact the housing at full velocity. The machine must also be designed to withstand such impacts.

In order to efficiently engineer the machine, it is necessary to understand the effects of impact on design features. At present, these effects are verified using a transient finite-element analysis (FEA).

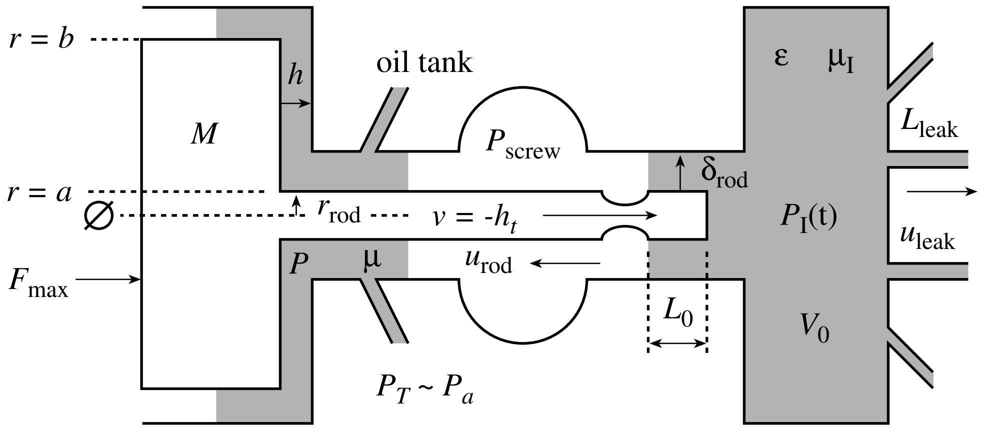

Figure 3.1: Schematic diagram of injection molding machine. See Section 3.4 for definitions of all variables and parameters.

However, an FEA is not only time consuming but the company's FEA resources are limited. Therefore, a simplified model, that could be used by a designer to obtain a first pass type of analysis, is desired. Once appropriate design features are obtained using the simplified model, they can be verified using an FEA.

Husky was interested in finding a simplified model of the forces involved in the impact of the screw on the molten metal, and the impact of the piston flange on the injection housing (i.e., when the piston "bottomed out"). For the impact of the screw, information is desired regarding the effect of the molten metal's bulk modulus (alternatively, its compressibility), and regarding the effect of the leakage flow rate of the molten metal past the impacting screw tip created by clearances between the screw and the housing (see $u_{\rm rod}$ in Figure 3.1). Currently, the company's analysis of the "bottom out" problem assumes a dry contact between the piston flange and injection housing, when, in fact, there is a thin film of hydraulic fluid between the impacting bodies that will significantly reduce the strain on the system. Information regarding the extent of this reduction is desired.

The company also wished to include the effects of material damping in the simplified models. However, we decided that an investigation of this effect would best be approached in an FEA. Therefore, given the limited time of the workshop, it was decided that we would postpone this investigation.

The model currently used by Husky ignores all effects except the deformation of the piston and housing, which is assumed to be elastic. The impact is modelled using a mass-spring system, where the force of impact is obtained for a given impact velocity, system mass and pre-calculated "spring" constants for the impacting bodies. In order to determine the importance of the presence of the hydraulic fluid, the compressibility of the molten metal, and the leakage past the screw tip, we will assume that the deformation of the piston and housing can be ignored. However, in order to make quantitative predictions of the impact forces, it is necessary to include the effects due to the compressibility of the machine parts. Thus, we also investigate a model that, like the current Husky model, uses a mass-spring system to incorporate the elastic deformation of the piston and the housing.

The goal is to derive simplified equations of motion for the piston that can be used to generate pressure profiles that occur during impact. The pressure profiles can be used to determine if the design features are sufficient to ensure that the impact is not likely to cause damage to the machine. In the next section, we consider the “bottom out” problem. In particular, we discuss the effect of the hydraulic fluid on the impact. In section 3.3, we consider the impact of the screw on the molten metal. In particular, the effects of the leakage of the molten metal past the screw tip and the effect of the compressibility of the molten metal are discussed. In section 3.4 results from a model that ignores the effects due to the deformation of the machine parts are presented. In section 3.5, we discuss preliminary results that have been obtained on a simple model that assumes elastic deformation of the piston and housing.

3.2 The “bottom out” problem

It is well-known that lubricants can support high loads in sufficiently small gaps. A lubricant will act to prevent contact between moving components and reduce stress by spreading the load. This is the reason that automobile or machine parts exhibit very little wear when adequately lubricated. The process may be modelled by the lubrication approximation to the Navier-Stokes equations.

When a two-dimensional flat indenter is pressed onto an elastic surface, a stress singularity occurs at the edges of the indenter (in practice, some plastic deformation will occur to reduce this). For this reason it is common practice to round edges and so prevent excessively high stresses. When a fluid is placed between the indenter and the elastic body the fluid forms what is known as a squeeze film. The pressure in the squeeze film is highest at the centre and reduces to the ambient pressure at the edges of contact. Obviously the stress distribution is completely different from the case when there is no fluid present, with the lubricated contact being much less likely to exhibit stress related wear.

Due to the presence of hydraulic fluid between the piston flange and the housing, this lubrication approximation may be employed in the “bottom out” problem. During the workshop, a model of the oil squeeze film between the flange and housing was developed. The process occurs in two stages, described by constant velocity and constant force. During the constant velocity phase, the squeeze film model may be used to predict the pressure distribution in the fluid. When the load (integral of the pressure) reaches the maximum operating value, the model switches to a constant load situation. An equation of motion was derived for the evolution of the film thickness during this second stage. However, the main aim of the analysis was to determine the force that is exerted by the squeeze film on the piston, and thus, a relation between the force and the film thickness is required. When the flange and housing are flat (or conform), such a relation can be found in the form of a simple analytic expression that describes the force as a function of film thickness (and the rate of change of film thickness).

3.3 The piston screw – molten metal impact

In considering the impact of the piston screw on the molten metal, it is expected that both the compressibility of the molten metal and the leakage of the molten metal past the screw tip will decrease the strain on the system.

To investigate these effects, a simplified model of the impact was derived from the equation for the conservation of mass of the molten metal. That is, we obtain an equation indicating that the rate of change of mass within the mold itself is equal to the rate at which the mass leaves the mold. The mass is written as density ρ times volume V of the mold (which includes all volume on the mold side of the screw tip). It is assumed that the density can be written as a function of the pressure in the mold, P_{I}, only, and that the pressure P_{I} is spatially constant. As is the case for the squeeze film, the process occurs in two stages, the first is a constant velocity stage, while the second is a constant load. As for the squeeze film, we assume that the stage of interest is the second, and that at the beginning of this stage the mold has been filled (no holes) and the initial velocity of the piston is the velocity that is prescribed during the first stage.

3.4 Results

The problem now reduces to a coupled system of differential equations with dependent variables h (representing both the height of the squeeze film and the position of the piston) and P_{I} (the pressure in the mold). The resulting equations are as follows:

$$\begin{aligned} M h_{tt} &= -\frac{6\mu h_t}{h^3} I + P_{\mathrm{I}} \pi r_{\mathrm{rod}}^2 - \frac{P_a}{2}\left(b^2 - a^2\right) - F_{\mathrm{app}} && (3.1) \\ \frac{\partial P_{\mathrm{I}}}{\partial t} &= \frac{1}{V_0 + \pi r_{\mathrm{rod}}^2 h} \frac{1}{\varepsilon}\left[-\pi r_{\mathrm{rod}}^2 h_t - \pi r_{\mathrm{rod}} \delta_{\mathrm{rod}} h_t - \frac{\pi r_{\mathrm{rod}} \delta_{\mathrm{rod}}^3 \left(P_{\mathrm{I}} - P_{\mathrm{screw}}\right)}{6 L_0 \mu_{\mathrm{I}}}\right] && (3.2) \end{aligned}$$

where h is the height of the squeeze film between the flange and housing, P_{I} is the pressure inside the mold, $V(t) = V_0 + \pi r_{\mathrm{rod}}^2 h$ is the volume of the molten metal inside the mold, ε is the compressibility of the molten metal, μ is the viscosity of the hydraulic fluid, μ_{I} is the viscosity of the molten metal, M is the mass of the piston (including the screw), F_{app} is the (constant) force applied to the piston during the second stage (i.e., $F_{\mathrm{app}} = F_{\mathrm{max}}$), P_{screw} is the pressure between the screw and the housing, P_a is the ambient pressure (pressure in the oil tank, see Figure 3.1), r_{rod} is the radius of the piston (the radius of the piston rod and the piston screw are assumed to be equal), δ_{rod} is the width of the gap between the screw and the injection housing, L_0 is the length of the screw, and

$$I = 2\pi \left[\frac{1}{2} b^4 \ln \frac{b}{a} - \frac{1}{8}\left(3b^2 - a^2\right)\left(b^2 - a^2\right)\right], \tag{3.3}$$

that is, a constant that depends on b and a, the radius of the flange (or the flange width) and the radius of the housing along the length of the piston rod, respectively. Equation (3.1) describes the change in height h of the squeeze film, and depends on two unknowns, h and the pressure in the mold P_{I}. The various terms on the right hand side show that the deceleration of the piston depends on the current film thickness, the pressure on the end of the rod, the ambient pressure and the applied force. Equation (3.2) describes the pressure within the metal. This depends on the rod motion (which depends on the squeeze film thickness) and the leakage between the rod and housing.

We non-dimensionalise using the following scaling factors:

$$h = h_0 x_1, \qquad h_t = \frac{h_0}{t_0} x_2, \qquad P_{\mathrm{I}} - P_{\mathrm{screw}} = P_0 x_3, \tag{3.4}$$

(i.e. x_1 is the scaled height of the squeeze film, x_2 is the scaled piston velocity, and x_3 is the scaled pressure in the mold) to obtain the equations of motion

$$x_1' = x_2 \tag{3.5}$$

$$x_2' = -\alpha \frac{x_2}{x_1^3} + \beta x_3 - f_{\text{app}} + \beta \frac{P_{\text{a}}}{P_0} \tag{3.6}$$

$$x_3' = \frac{1}{1+\delta x_1} \left[-x_2 \left(1 + \frac{\delta_{\text{rod}}}{r_{\text{rod}}} \right) - \gamma x_3 \right] \tag{3.7}$$

where t_0 is defined as h_0/v_{init}, v_{init} is the initial velocity of the piston (i.e., the velocity prescribed in the first stage of the impact). In addition,

$$P_0 = \frac{M v_{\text{init}}^2}{h_0 \pi r_{\text{rod}}^2} = \frac{1}{\varepsilon} \frac{\pi r_{\text{rod}}^2 h_0}{V_0}, \tag{3.8}$$

where the first equality describes the pressure required to stop the moving piston in a distance h_0, while the second equality describes the pressure induced by compressing the molten metal a distance h_0 (equating these can produce an expression for h_0). The system is controlled by six dimensionless parameters, the aspect ratio $\delta_{\text{rod}}/r_{\text{rod}}$, (i.e. the ratio of the gap width to the radius of the piston rod), and five others α, β, γ, δ, and f_{app}, which are related to the physical parameters as follows:

$$\alpha = \frac{6 I \mu v_{\text{init}}/h_0^2}{M v_{\text{init}}^2} \tag{3.9}$$

i.e. α is the ratio of the energy dissipated in the squeeze film to the initial energy,

$$\beta = \frac{P_0 h_0 \pi r_{rod}^2}{M v_{\text{init}}^2} = \frac{\frac{\Delta V}{\varepsilon} \frac{\Delta V}{V_0}}{M v_{\text{init}}^2} \tag{3.10}$$

(where $\Delta V = \pi r_{\text{rod}}^2 h_0$ is the change in the volume V of molten metal in the mold corresponding to a change in piston position h of h_0), i.e. β is the ratio of the energy required to compress the molten metal a distance h_0 to the initial energy, which by definition implies that $\beta = 1$,

$$\gamma = \frac{P_0 t_0 \delta_{\text{rod}}^3}{6 r_{\text{rod}} h_0 L_0 \mu_{\text{I}}} = \frac{2\pi \delta_{\text{rod}} r_{\text{rod}} \frac{P_0 \delta_{\text{rod}}^2}{12 \mu_{\text{I}} L_0}}{\pi r_{\text{rod}} v_{\text{init}}} \tag{3.11}$$

i.e. γ is the ratio of the leakage flow rate at maximum compression to the initial flow rate,

$$\delta = \frac{h_0 \pi r_{\text{rod}}^2}{V_0} \ll 1 \tag{3.12}$$

i.e. δ is the ratio of volume of compression of molten metal to volume of the mold, and

$$f_{\text{app}} = \frac{t_0^2 F_{\text{app}}}{h_0 M} \tag{3.13}$$

is the non-dimensionalised applied (constant) force. Because δ is very small, it may be neglected and therefore, there are only five parameters that control the motion of the piston. Thus, working with the non-dimensional equations reduces the number of parameters that need be considered. For example, from the definition of the dimensionless parameter α it can be seen that increasing the viscosity of the hydraulic fluid μ will have the same effect as increasing I by increasing the flange width b. Alternatively, to investigate the effect of changing a physical quantity, we need only vary the dimensionless parameters accordingly. For example, we can increase the dimensionless parameter α if we wish to determine the effect of increasing the flange width b.

We now carry out some preliminary numerical calculations to demonstrate the various possible types of behaviour that can be observed. For example, we expect that if there is little leakage of molten metal past the screw tip, then the piston will be stopped by the molten metal. However, if there is significant leakage, then it will be the squeeze film that acts to stop the motion. In the first example, we choose the gap width between the screw and the injection housing $\delta_{\text{rod}} = 50\mu$m, which is a reasonably large gap through which the molten metal may pass. The results of the numerical calculations, shown in Figure 3.2, indicate that the load of the impact is shared by both the squeeze film and the molten metal (although a large part of the load is taken by the squeeze film; see below). The squeeze film height h initially decreases rapidly and then slowly tends to a constant (non-zero) value. The velocity of the piston has a corresponding initial stage when it changes slowly, and then a stage when it rapidly tends to zero. The scaled pressure starts at a low value and then increases rapidly as impact is approached. As the piston velocity decreases, the pressure decreases until it reaches a low value sufficient to balance the applied force.

As the gap width δ_{rod} decreases, there is less leakage and more of the load is taken by impact of the screw on the molten metal. Calculations for the case when the gap width δ_{rod} is 25μm are shown in Figure 3.3. As in the previous case, the squeeze film height h initially decreases rapidly, however, the relaxation of the film height h to the constant value is slower, and the deceleration of the piston is not as sharp. Of particular interest is that an increase in the maximum of P_{I} (the pressure in the mold) is observed. For the calculations, the pressure parameter P_0, given in (3.8), is taken to be 3.13×10^8Pa, which implies that for the case of the gap width $\delta_{\text{rod}} = 50\mu$m there is a maximum pressure of approximately 6×10^7Pa, while for the case of the gap width $\delta_{\text{rod}} = 25\mu$m there is a maximum pressure of approximately 4 times this (2.4×10^8Pa).

If the gap width δ_{rod} is taken to be as small as 10μm (very little leakage), there seems to be a qualitative change in the solution; see Figure 3.4. In this case, the initial kinetic energy of the piston goes almost fully into compressing the molten metal, i.e. the squeeze film takes little of the burden. This compression is assumed to be elastic, and thus, because there is very little leakage, the piston rebounds, and oscillates. Note that there is a very slight rebound in the case of the gap width $\delta_{\text{rod}} = 25\mu$m, so this effect is already being seen here.

The definition (3.9) of the dimensionless parameter α indicates that as α increases, the effects of the squeeze film become more important. In particular, in the case when both the dimensionless parameter α and the gap width δ_{rod} are large, the full impact of the piston is absorbed by the squeeze film, i.e., this corresponds to the situation when there is no molten metal in the mold. The parameter α can be increased by increasing the value of I (see equation 3.3) which in turn can be increased by increasing the flange width b. The results of increasing the dimensionless parameter α can be seen in Figure 3.5. Qualitatively, the behaviour is similar to the situation shown in Figure 3.2, except that there is a sharper deceleration of the piston on impact.

Figure 3.6 contains a plot comparing the pressure in the mold P_{I} to the spatial maximum pressure in the squeeze film as a function of time for the case when the gap width $\delta_{\text{rod}} = 50\mu$m, and the flange

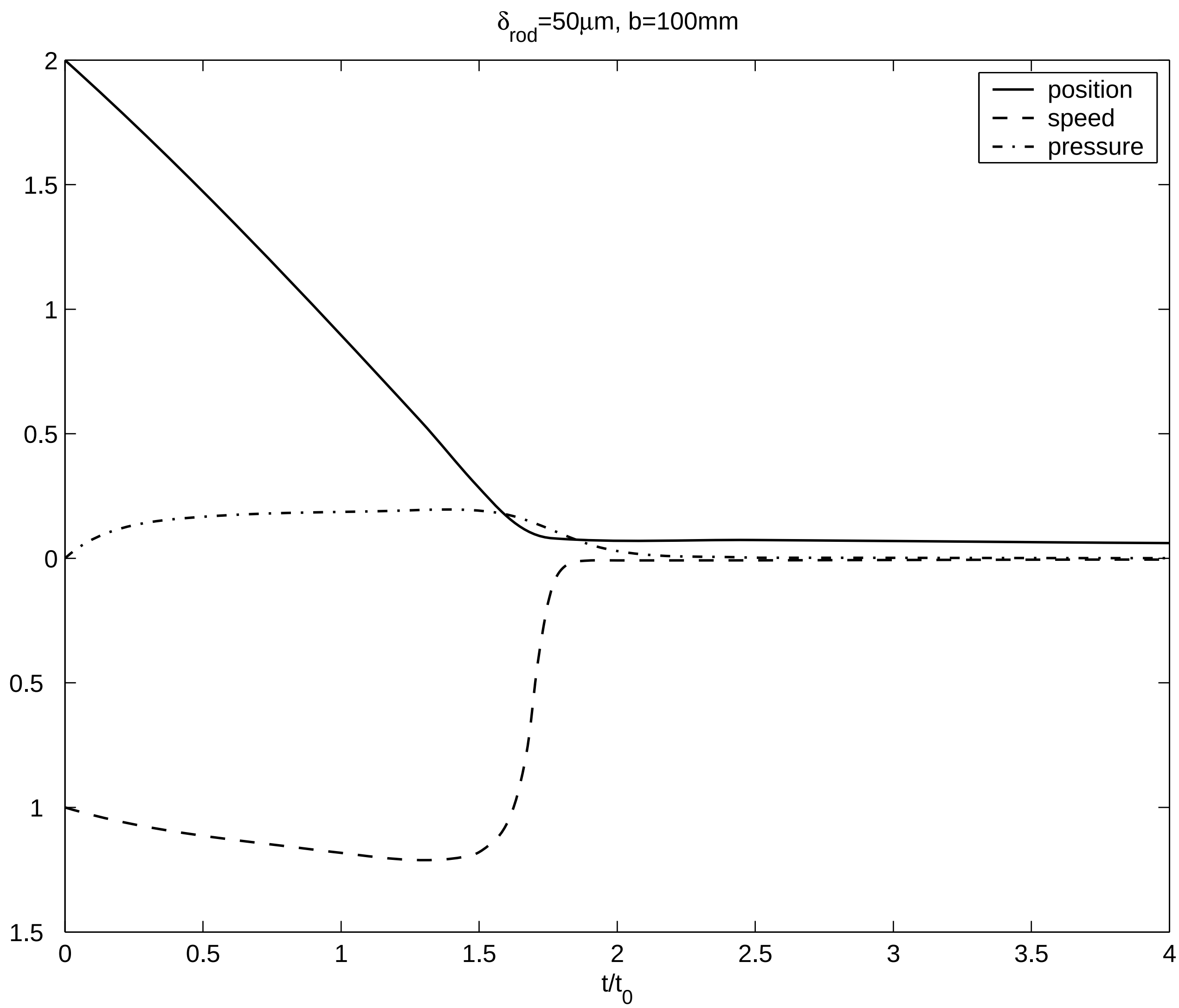

Figure 3.2: Results of simulation with moderate leakage: the gap width $\delta_{\text{rod}} = 50\mu$m. Position, speed and pressure refer to the non-dimensionalised variables, x_1, x_2 and x_3, respectively. These related to the dimensional quantities via (3.4).

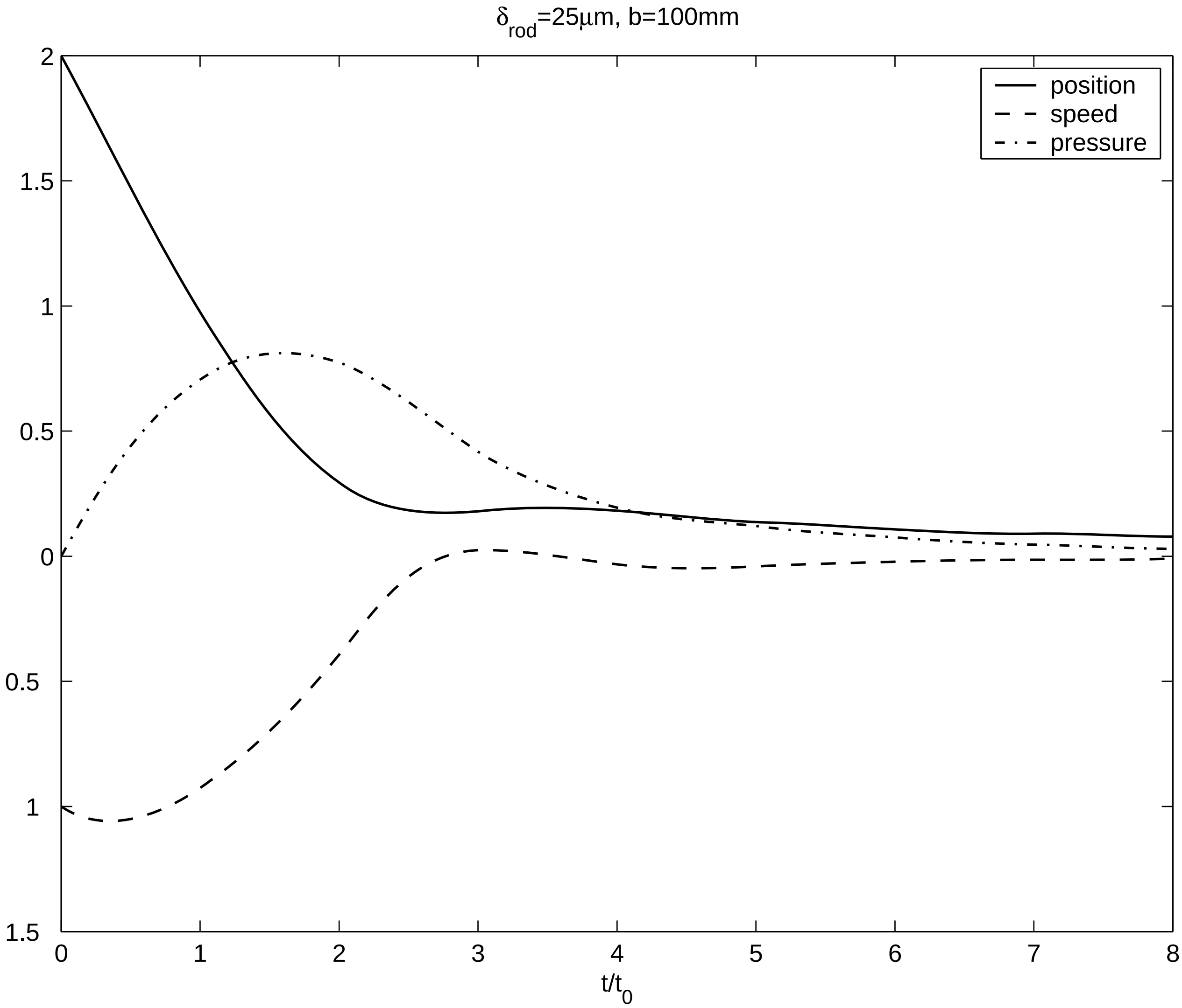

Figure 3.3: Results of simulation with small leakage: the gap width $\delta_{\rm rod} = 25\mu$m. Position, speed and pressure refer to the non-dimensionalised variables, x_1, x_2 and x_3, respectively. These related to the dimensional quantities via (3.4).

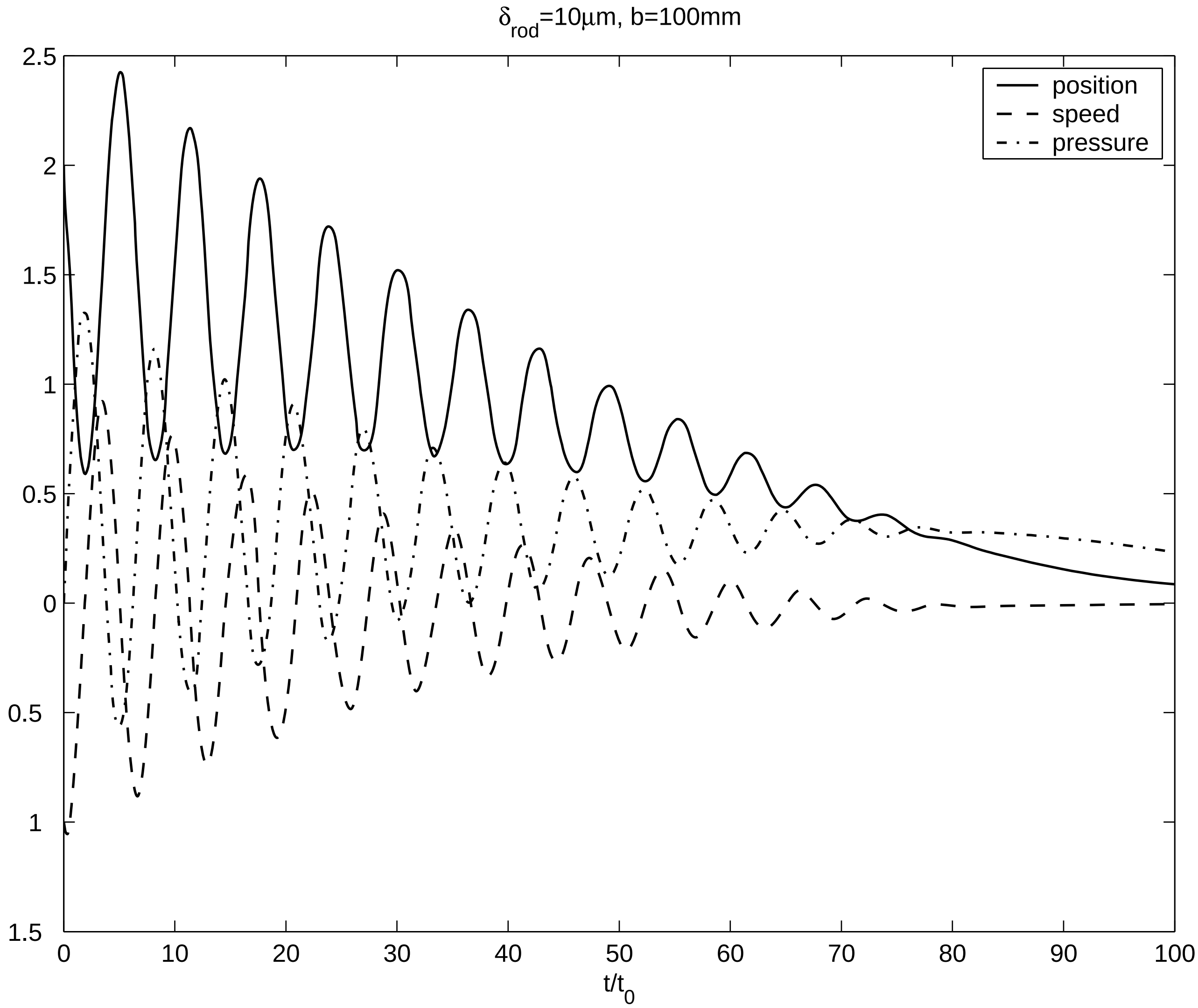

Figure 3.4: Results of simulation with extremely small leakage: the gap width $\delta_{\rm rod} = 10\mu$m. Position, speed and pressure refer to the non-dimensionalised variables, x_1, x_2 and x_3, respectively. These related to the dimensional quantities via (3.4).

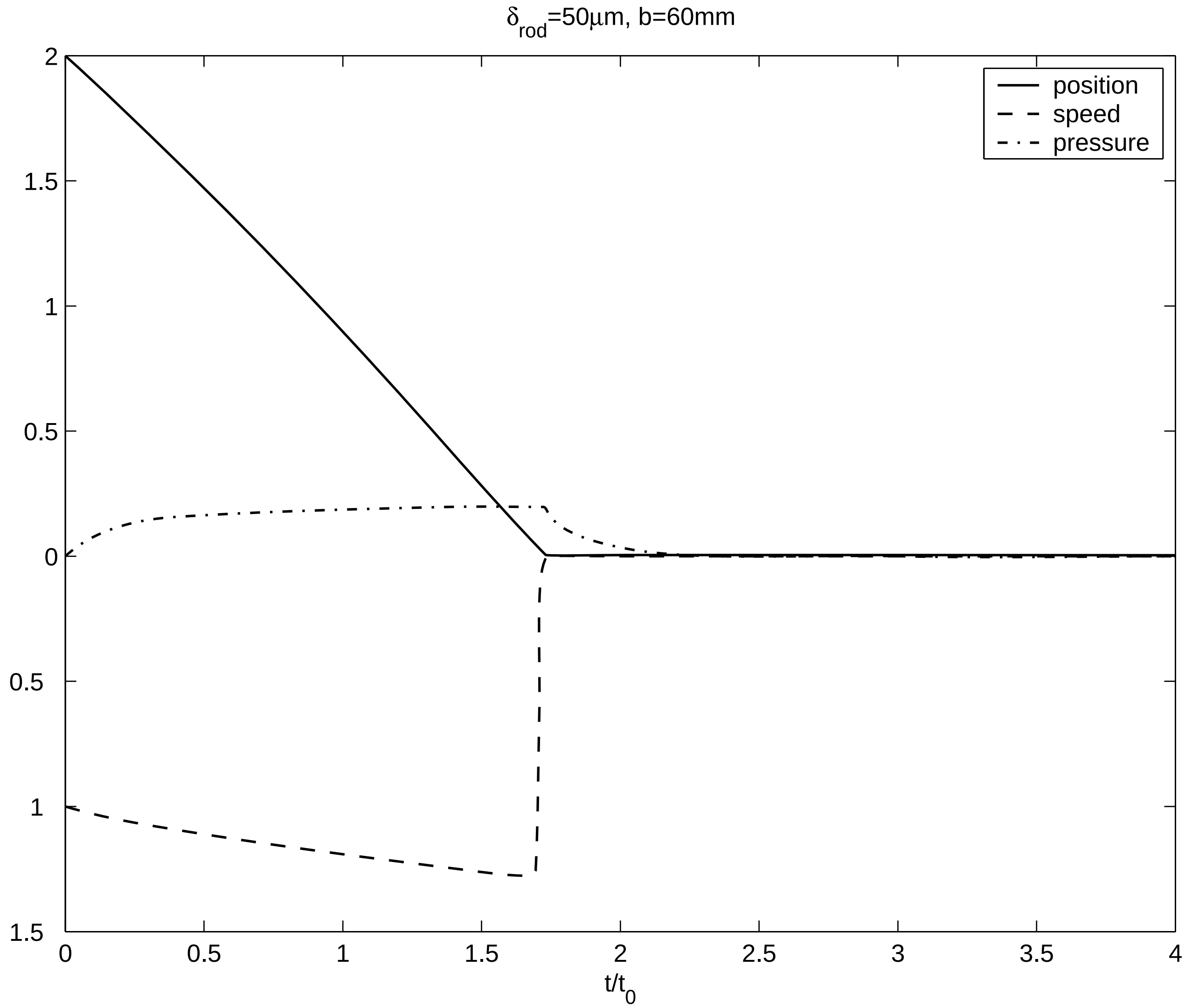

Figure 3.5: Results of simulation with moderate leakage: the gap width $\delta_{\mathrm{rod}} = 50\mu$m, and the flange width $b = 60$mm. Position, speed and pressure refer to the non-dimensional variables, x_1, x_2 and x_3, respectively. These related to the dimensional quantities via (3.4).

width $b = 100$mm. As mentioned above the maximum of P_I, the pressure in the mold, in dimensional units is approximately 6×10^7Pa. The maximum pressure in the squeeze film is nearly 10 times greater. This pressure is felt over only a small area near the outside of the flange; the pressure decreases rapidly toward the inner part of the flange. To illustrate this, the pressure in the squeeze film, as both a function of time and distance along the flange, is plotted in Figure 3.7.

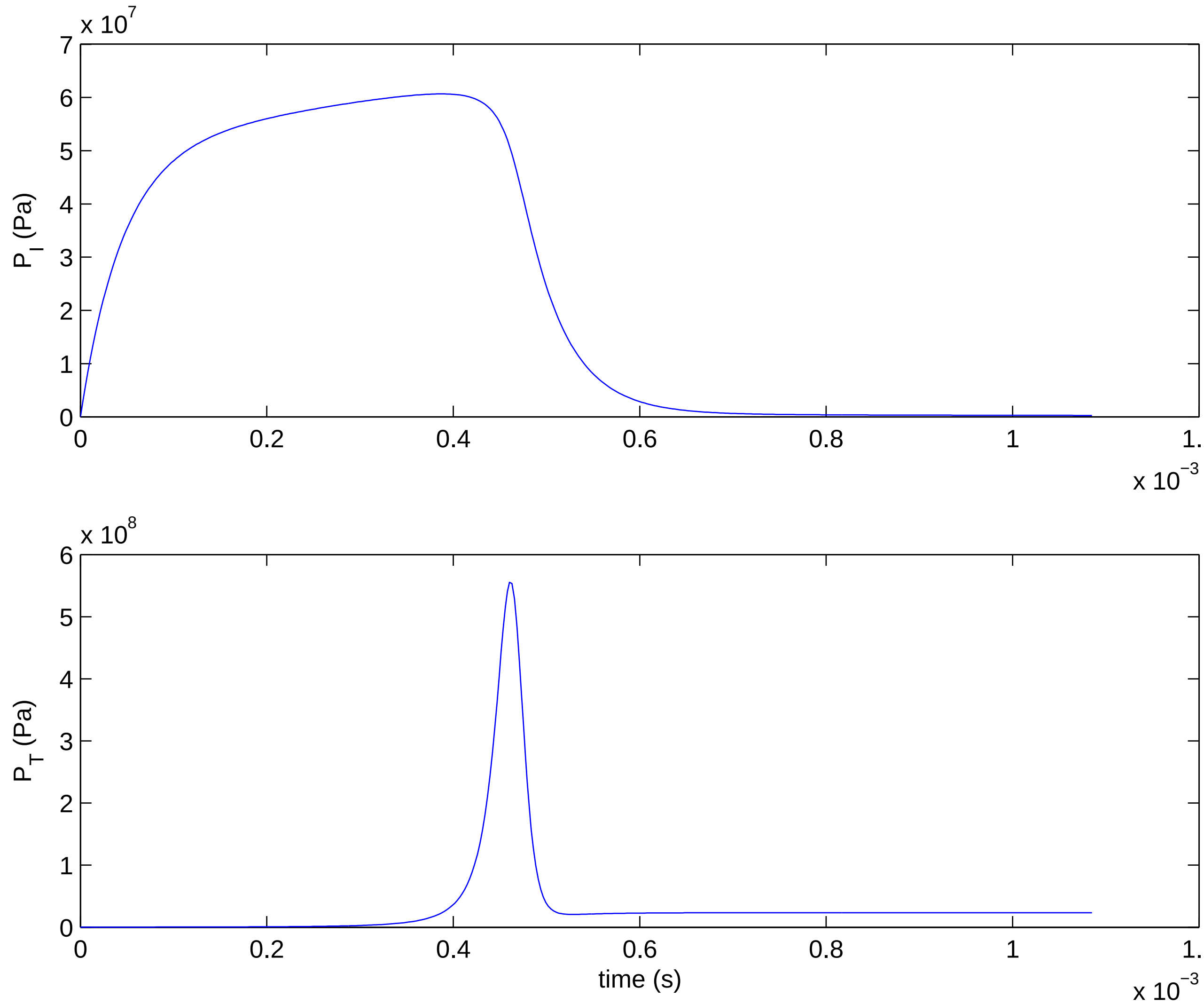

Figure 3.6: Pressure in mold P_I and pressure (spatial maximum) in squeeze film P_T vs. time for the parameter values the gap width $\delta_{\mathrm{rod}} = 50\mu$m, and $b = 100$mm.

3.5 Effects of compressibility of piston and housing

The results of the previous section indicate that depending on the circumstances, either the squeeze film or the molten metal can act to stop the piston, and therefore both should be included in any math-

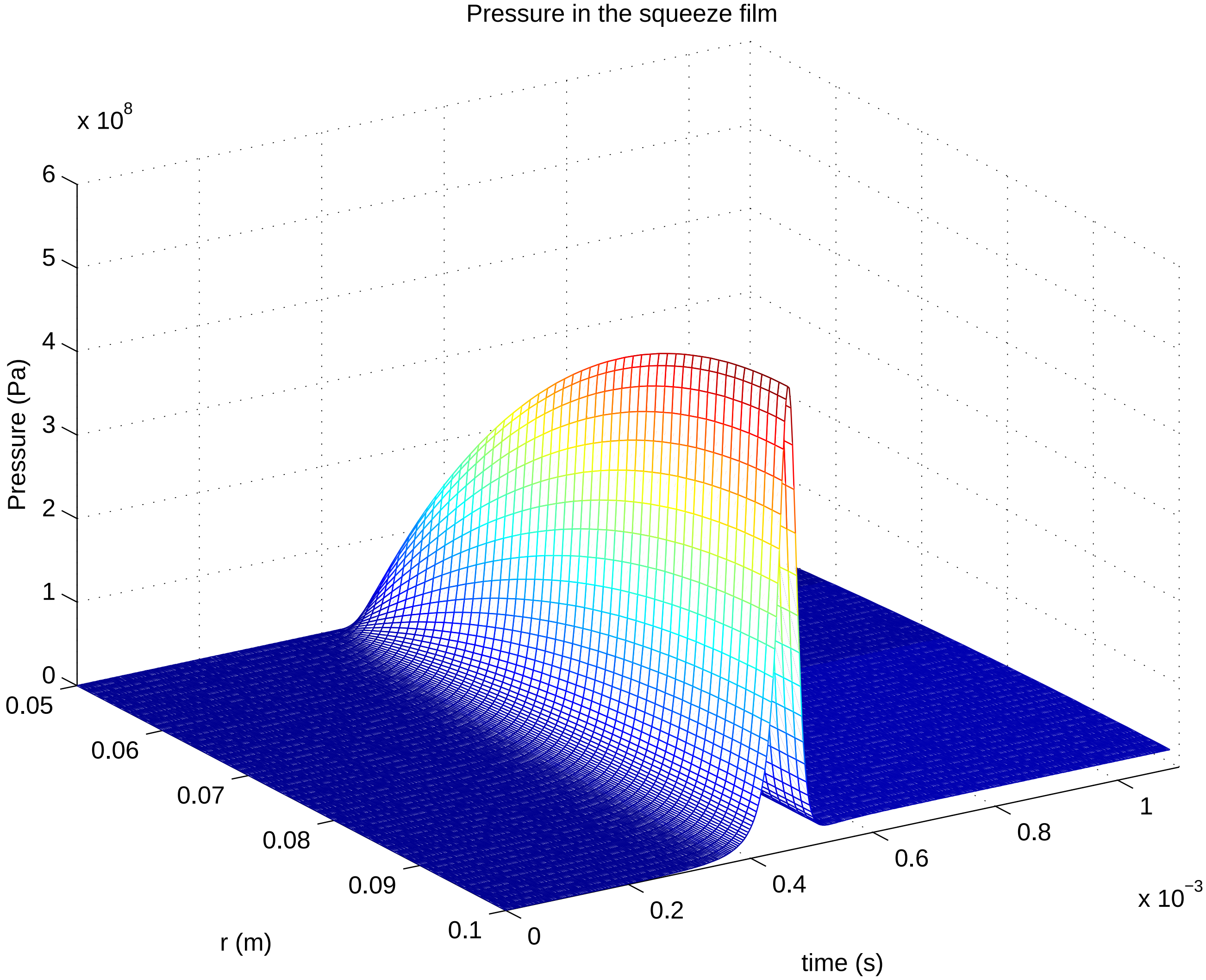

Figure 3.7: Pressure in squeeze film vs. time and radius.

ematical model of the process. The actual numerical values of pressure that were obtained, however, cannot be taken as the values that would occur in practice. At the pressures that are predicted, the compressibility of the piston and housing would likely have an effect.

Therefore, a second model was developed to investigate the effects of the elasticity of the machine parts on the impact of the piston flange on the housing. We consider a one dimensional model that is represented in Figure 3.8. The piston is modelled as two bodies of mass M_1 and M_2 coupled together by a spring with spring constant k_1 and uncompressed length l_1, and the housing is modelled by another body of mass M_3 that is attached to an immovable body (wall) by a spring with spring constant k_2 and uncompressed length l_2. The piston moves toward the housing, with the second body M_2 impacting the housing. It is assumed that hydraulic fluid is present between the two impacting bodies, and thus, a squeeze film is created during impact. As pressure is generated in the squeeze film, the springs associated with the piston and housing begin to compress. All interactions are assumed to be elastic.

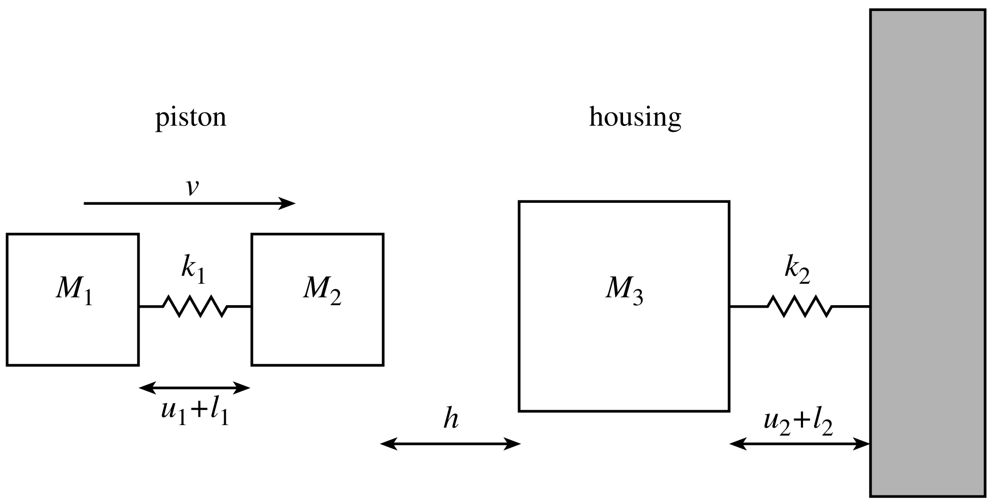

Figure 3.8: Model system of impact of piston and housing including both the effects of the squeeze film and the effects of the elasticity of the machine parts.

The model consists of the equations of motion for the three bodies where the spring forces, the force exerted by the squeeze film and the applied force are included. The equations describe the evolution of the three variables: h the distance between the piston and housing, u_1 the distance that the piston spring is compressed (that is, $u_1 + l_1$ is the distance between M_1 and M_2), and u_2 the distance that the spring of the housing is compressed. See Figure 3.8.

As in the previous section, we carry out some preliminary calculations. The case when $k_1 = 2 \times 10^9$kg/sec^2 and $k_2 = 7.2 \times 10^9$kg/sec^2 is plotted in Figures 3.9. Other relevant parameters are taken to be the same as the case plotted in Figure 3.2. Initially, the housing is stationary and the spring of the piston is not compressed. As the piston approaches the housing, the effects of the squeeze film begin to be felt and the spring of the piston begins to be compressed. As the piston impacts the housing, the squeeze film pressure spikes, with corresponding jumps in the rates of change of the variables. After impact, the piston and housing essentially become stuck together (very little variation of h, the distance between the piston and housing, from zero), and there is an oscillation of all other variables. Of particular interest is the peak pressure that is observed in the squeeze film. In this case, the maximum pressure is less than 2×10^7Pa, while in the situation where the elasticity of the machine parts is not considered, the maximum pressure is more than 5×10^8Pa. That is, the elasticity reduces impact pressure by more than an order of magnitude. We have not yet investigated how this result changes as parameters are varied. This will be discussed further in the final report.

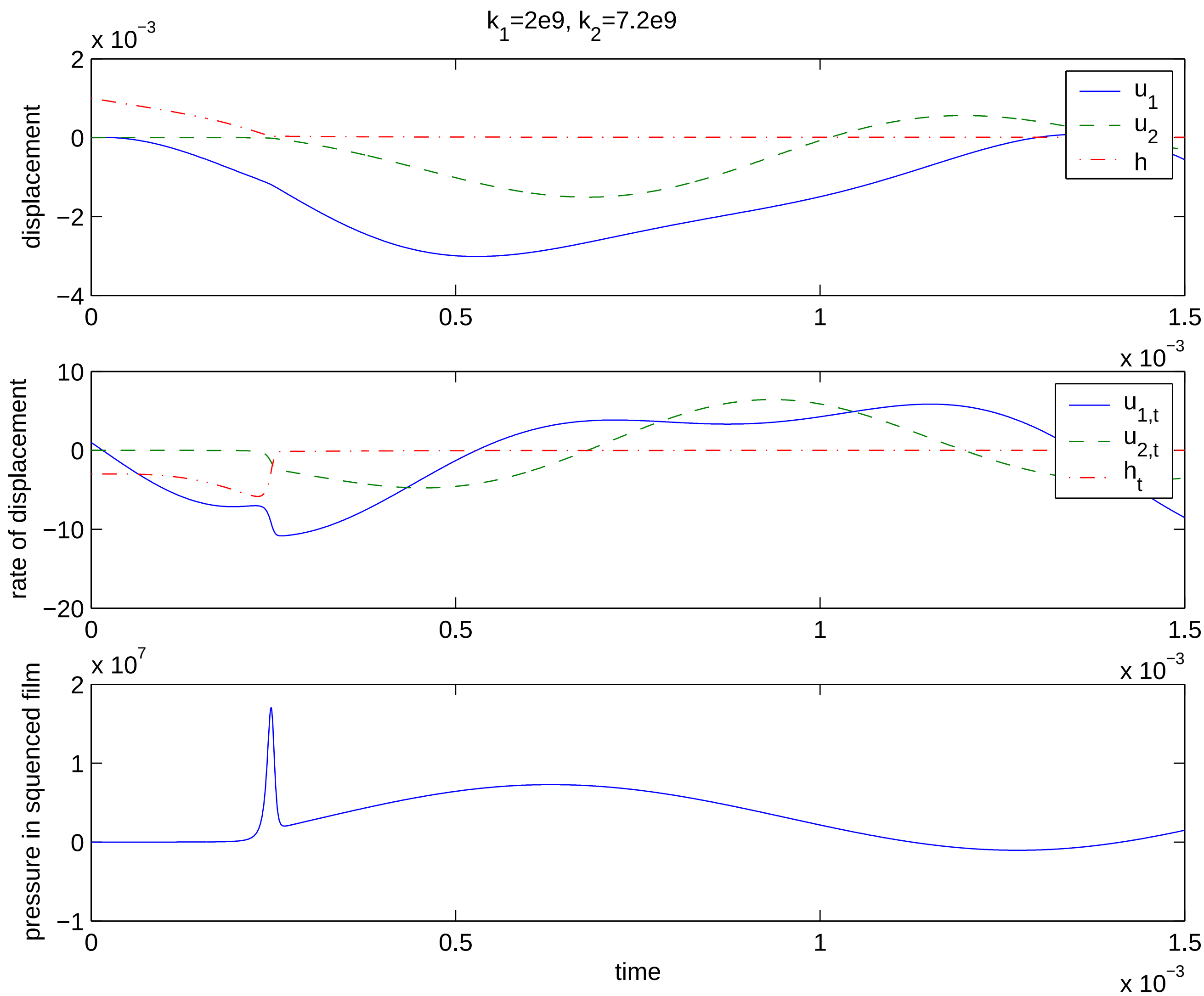

Figure 3.9: Results from model of impact of piston and housing including both the effects of the squeeze film and the effects of the elasticity of the machine parts. $k_1 = 2 \times 10^9$kg/sec^2 and $k_2 = 7.2 \times 10^9$kg/sec^2, and other constants are taken to be those for the case plotted in Figure 3.2.

3.6 Concluding Remarks

The results indicate that the presence of the hydraulic fluid, the compressibility of the machine parts and molten metal, and the leakage of molten metal past the screw tip can all produce significant effects on the pressure profiles and therefore should all be included in any mathematical model of the process.

In the final report, the models and results that are briefly presented here will be described in detail. The discussion will include the derivation of the forces generated in the squeeze film and the mold, and will also include more details of the model that assumes elasticity of the machine parts. Any improvements in the models that are made will also be reported. In particular, an improved (continuous) model that incorporates all the effects discussed here, including the compressibility of the machine parts, is currently being developed.