Strategic Planning at Kruger Products

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1. Introduction

Kruger Products is a major provider of tissue consumer products to the North American market with leading consumer brands such as Cashmere, Sponge Towels, and Scotties. A major challenge for the company is to make decisions on where and when major investments in capital should be made. Currently, these decisions are based on estimates of costs and revenues that do not take into account the whole logistics network and its evolution over time. Therefore, the company would like to acquire a strategic planning tool that would help it decide whether to expand existing plants, acquire new ones, or close existing ones.

It is in this context that John O'Hara from Kruger Products and Vincent Béchard from Différence S.E.N.C. (a consultant for Kruger Products) submitted the strategic planning problem at Kruger Products to the First Montreal Industrial Problem Solving Workhsop (August 20-24, 2007), organized by the CRM and financed by MITACS and the ncm_2. Along with Bernard Gendron and Jean-François Cordeau (two researchers specialized in operations reasearch applied to logistics and transportation), nine students worked on various aspects of the problem during the workshop: Stefan Ropke, Mervat Chouman, Mehdi El Ouali, Xiaorui Fu, Hamid Ghaffari, Paul-Virak Khuong, Yang Li, Xi Liu, and Robert Warren.

During the workshop, the following activities were performed by the team.

- *Modeling the problem.* Following an initial discussion with the industrial partner on the first day of the workshop, a formal definition of the problem was proposed by the participants and validated by the industrial partner. A first mathematical model, called the *Basic Model*, was then formulated. During that time, some students performed a literature review on related strategic planning problems. Although the Basic Model is an accurate representation of the problem, it has potential deficiencies, as demonstrated by existing studies on similar problems. Hence, two alternative models, the *Disaggregated Model* and the *Path Model*, were proposed.

- *Developing computer codes.* The industrial partner did not provide any real-world instances. We were able, however, to propose a data file format based on the description of the problem, in order to allow the team to test solution methods on toy problems. One student also developed a data generator that randomly constructs instances according to this data file format. Two students programmed a C++ code implementing the Basic Model; their code uses the ILOG Concert Library and calls the state-of-the-art mixed integer programming (MIP) solver ILOG CPLEX. Another student implemented the Basic Model using the modeling tool AMPL, which generates input files for CPLEX. Due to a lack of time, these codes could not be tested.

- *Designing solution methods.* For realistic instances, the Disaggregated and Path models yield very large-scale formulations, which cannot be solved directly by a MIP solver. Hence the researchers and postdoctoral students started to investigate decomposition methods for these models, based on cutting-plane, Lagrangian relaxation, and column generation approaches.

The rest of this report is organized as follows. In Section 2, we give a description of the strategic planning problem at Kruger Products. Section 3 first introduces the notation used in the different formulations and then presents the Basic Model. Section 4 and Section5 describe the Disaggregated and Path Models, respectively. We conclude this report by outlining avenues for future work.

2. Problem Description

The strategic planning problem at Kruger Products can be represented using a two-echelon logistics network comprising three nodes sets: the suppliers, the plants, and the customers. The only possible connections link suppliers to plants and plants to customers. The suppliers represent the sites that provide wood (or raw material) to the plants, which are either existing ones or potential facilities. The wood is processed at the plants to generate the finished products. There is a preliminary processing stage at the plants that creates intermediate products; those can be processed at the same plant or delivered to another plant in order to be transformed into finished products. At each stage, we know how many units of raw material (resp. intermediate product) are used to produce one unit of intermediate product (resp. final product).

The company seeks to optimize its investments over a period of ten years, with new decisions about the plants being evaluated every year. The company wishes to enforce the constraint that a plant that is closed (resp. open) at the beginning of the planning horizong and is then opened (resp. closed) at some period remains so for the rest of the planning horizon.

For each year of the planning horizon, we assume that we have estimates of the demand of each customer for each finished product, as well as capacities and lower bounds at the suppliers and at the plants, both global and for each type of product (raw material or intermediate product). There are fixed costs for opening and operating the potential plants, but also for supplying some amount of raw material. The variable costs consist of three components: manufacturing costs, transportation costs, and benefits from sales. The latter being larger than the sum of the first two, we can capture the variable cost for each link-product combination at each period by using a generic, negative, variable cost.

Given these fixed and variable costs, the problem consists of minimizing the resulting objective function, such that demands and production bounds are satisfied over a planning horizon of ten years, with the additional constraint that any plant that is opened (or closed) at some point during the planning horizon remains so for the rest of the horizon.

3. Notation and Basic Model

The notation used to model the problem and the three groups of variables are given in the following tables. To obtain a compact representation of the constraints, we consider all types of products (raw material, intermediate or finished products) as members of a generic commodities set. Each commodity has several origins and several destinations, where an origin-destination pair corresponds to a link in the network. To model the situation where intermediate and finished products are generated at the same plant, we create two instances of each plant with a link between the two instances.

Symbol	Description
	Sets
S	Set of suppliers.
P	Set of plant locations.
C	Set of customers.
R	Set of raw materials.
Q	Set of intermediate products.
F	Set of finished products.
T	Set of time periods.
$O = S \cup P$	Set of origin nodes.
$D = P \cup C$	Set of destination nodes.
$K = R \cup Q \cup F$	Set of all commodities.
$O^k \subseteq O$	Set of origins that provide commodity $k \in K$.
$D^k \subseteq D$	Set of destinations that require commodity $k \in K$.
$P^k \subseteq P$	Set of plants that potentially require commodity $k \in K$ to make their products.
$K^o \subseteq K$	Set of commodities that are produced at origin $o \in O$.
$P_1 \subseteq P$	Set of plants that are open at the start of the planning horizon.
$P_0 \subseteq P$	Set of plants that are closed at the start of the planning horizon.
	Data
a^t_{cf}	Max. demand of customer $c \in C$ for product $f \in F$ in period $t \in T$.
$\overline{q}^t_{ok}$	Maximum production of commodity $k \in K$ at origin $o \in O$ in period $t \in T$.
$\underline{q}^t_{ok}$	Minimum production of commodity $k \in K$ at origin $o \in O$ in period $t \in T$ if origin o is selected.
$\overline{u}^t_o$	Overall capacity at origin $o \in O$ in time period $t \in T$.
$\underline{u}^t_o$	Minimum production at origin $o \in O$ in time period $t \in T$ if origin o is selected.

Symbol	Description
Data	
b_{pkj}	Number of units of $k \in K$ needed to have one unit of $j \in K$ at plant $p \in P$.
c_o^t	Fixed cost of selecting origin $o \in O$ in time period $t \in T$.
c_{ok}^t	Fixed cost of producing commodity $k \in K$ at origin $o \in O$ in time period $t \in T$.
c_{odk}^t	Cost of transporting one unit of commodity $k \in K$ from $o \in O$ to $d \in D$ in time period $t \in T$. If $d \in C$ then the cost also "includes" the revenue of selling one unit of commodity k to customer d. Thus c_{odk}^t can be negative when $d \in D$. The cost also includes the cost of producing one unit of commodity k if $o \in P$ and the cost of buying one unit of commodity k if $o \in S$.
L_p^k	Set of products that use commodity k at plant p.
Decision variables	
$y_o^t \in \{0,1\}$	equals 1 iff origin $o \in O$ is selected in period $t \in T$.
$v_{ok}^t \in \{0,1\}$	equals 1 iff origin $o \in O$ provides commodity $k \in K$ in period $t \in T$.
$x_{odk}^t \in \mathbb{R}^+$	Quantity of commodity $k \in K$ transported from origin $o \in O$ to destination $d \in D$ in period $t \in T$.

The variables representing the commodities correspond to links (or arcs) of the network. The resulting model is a multiperiod multicommodity generalized arc-based network flow formulation.

$$\min \sum_{t\in T}\sum_{o\in O} c_o^t y_o^t + \sum_{t\in T}\sum_{k\in K}\sum_{o\in O^k} c_{ok}^t v_{ok}^t + \sum_{t\in T}\sum_{k\in K}\sum_{o\in O^k}\sum_{d\in D^k} c_{odk}^t x_{odk}^t \quad (1)$$

subject to

$$\sum_{p\in P} x_{pcf}^t \le a_{cf}^t \qquad \forall c \in C, \forall t \in T, \forall f \in F \quad (2)$$

$$\sum_{o\in O^k} x_{opk}^t - \sum_{j\in L_p^k}\sum_{d\in D^j} b_{pkj} x_{pdj}^t = 0 \qquad \forall k \in R \cup Q, \forall p \in P^k, \forall t \in T \quad (3)$$

$$\sum_{k\in K^o}\sum_{d\in D^k} x_{odk}^t \le \overline{u}_o^t y_o^t \qquad \forall o \in O, \forall t \in T \quad (4)$$

$$\sum_{k\in K^o}\sum_{d\in D^k} x_{odk}^t \ge \underline{u}_o^t y_o^t \qquad \forall o \in O, \forall t \in T \quad (5)$$

$$\sum_{d\in D^k} x_{odk}^t \le \overline{q}_{ok}^t v_{ok}^t \qquad \forall o \in O, \forall k \in K^o, \forall t \in T \quad (6)$$

$$\sum_{d\in D^k} x_{odk}^t \ge \underline{q}_{ok}^t v_{ok}^t \qquad \forall o \in O, \forall k \in K^o, \forall t \in T \quad (7)$$

$$y_o^{t+1} \ge y_o^t \qquad \forall o \in P_0, \forall t \in T \quad (8)$$

$$y_o^{t+1} \le y_o^t \qquad \forall o \in P_1, \forall t \in T \quad (9)$$

$$y_o^t \in \{0,1\} \qquad \forall o \in O, \forall t \in T \quad (10)$$

$$v_{ok}^t \in \{0,1\} \qquad \forall o \in O, \forall t \in T, \forall k \in K \quad (11)$$

$$x_{odk}^t \in \mathbb{R}^+ \qquad \forall o \in O, \forall d \in D, \forall t \in T, \forall k \in K \quad (12)$$

Inequalities (2) enforce the customer demand constraint. Equalities (3) are the flow conservation constraints. Inequalities (4) and (5) enforce upper and lower bounds on overall production at origin $o \in O$. Inequalities (6) and (7) enforce upper and lower bounds on production of each commodity $k \in K$ at origin $o \in O$. Inequalities (8) ensure that once a plant has been opened, it is kept open for the rest of the planning horizon. Inequalities (9) ensure that once a plant has been closed, it is kept closed for the rest of the planning horizon.

4. Forcing Constraints and Disaggregated Model

The Basic Model is an accurate representation of the problem, but its linear programming (LP) relaxation is weak, especially when fixed charges are high and capacities are tight. Consequently, solving realistic instances by traditional LP-based branch-and-bound methods seems to be a difficult task. To improve the model, one might add valid inequalities, which for fixed-charge models often take the form of so-called *forcing constraints*, linking fixed-charge binary variables to other variables: thus the latter are forced to take the value 0 whenever a corresponding fixed-charge variable is equal to 0.

The first obvious forcing constraints relate the two families of binary variables:

$$v_{ok}^t \leq y_o^t \qquad \forall o \in O, \forall k \in K^o, \forall t \in T. \tag{13}$$

We can also define forcing constraints relating the flow of finished products at each customer and the binary variables:

$$x_{pcf}^t \leq a_{cf}^t v_{pf}^t \qquad \forall c \in C, \forall f \in F, \forall p \in O^f, \forall t \in T. \tag{14}$$

Similar forcing constraints can be defined for arcs between plants or between a supplier and a plant, but these valid inequalities are weak, since their right hand-side is the sum of the demands of finished products for several customers. We can obtain stronger forcing constraints by introducing disaggregated flow variables binding together the original flow variables and the destination of the flow (i.e., a "customer-finished product" combination). More precisely, for arcs between two plants, we define the variables x_{lpqcf}^t, where x_{lpqcf}^t denotes the "amount of intermediate product $q \in Q$ transported from plant $l \in P$ to plant $p \in P$ and entering into the composition of final product $f \in F$ delivered to customer $c \in C$ at time $t \in T$".

These variables are related to the original flow variables as follows:

$$\sum_{l \in P} \sum_{q \in Q} (b_{pqf})^{-1} x_{lpqcf}^t = x_{pcf}^t \qquad \forall c \in C, \forall f \in F, \forall p \in O^f, \forall t \in T. \tag{15}$$

We can then add the following forcing constraints:

$$x_{lpqcf}^t \leq (b_{pqf} a_{cf}^t) v_{lq}^t \qquad \forall c \in C, \forall f \in F, \forall p \in O^f, \forall q \in Q, \forall l \in O^q, \forall t \in T. \tag{16}$$

For arcs between suppliers and plants, we define the variables $x_{slrpqcf}^t$, where $x_{slrpqcf}^t$ denotes the "amount of raw material $r \in R$ transported from supplier $s \in S$ to plant $l \in P$, transformed into intermediate product $q \in Q$, then transported to plant $p \in P$ and entering into the composition of final product $f \in F$ delivered to customer $c \in C$ at time $t \in T$".

These variables are related to the disaggregated flow variables between plants by the following equations:

$$\sum_{s \in S} \sum_{r \in R} (b_{lrq})^{-1} x_{slrpqcf}^t = x_{lpqcf}^t \qquad \forall c \in C, \forall f \in F, \forall p \in O^f, \forall q \in Q, \forall l \in O^q, \forall t \in T. \tag{17}$$

We can then derive the forcing constraints below.

$$x^t_{slrpqcf} \leq (b_{lrq} b_{pqf} a^t_{cf}) v^t_{sr} \qquad \forall c \in C, \forall f \in F, \forall p \in O^f, \forall q \in Q, \; \forall l \in O^q, \forall r \in R, \forall s \in S^r, \forall t \in T \tag{18}$$

Note that S^r is the set of suppliers that can supply the commodity r. The disaggregated model is obtained by adding constraints (13)-(18) to the basic model. Using equations (17) and (15), it is possible to remove from the model all flow variables except the variables $x^t_{slrpqcf}$; indeed, the following equations express each type of flow variable as a function of the $x^t_{slrpqcf}$.

$$x^t_{pcf} = \sum_{s \in S} \sum_{r \in R} \sum_{l \in P} \sum_{q \in Q} (b_{lrq} b_{pqf})^{-1} x^t_{slrpqcf} \qquad \forall c \in C, \forall f \in F, \forall p \in O^f, \forall t \in T \tag{19}$$

$$x^t_{lpq} = \sum_{s \in S} \sum_{r \in R} \sum_{c \in C} \sum_{f \in F} (b_{lrq})^{-1} x^t_{slrpqcf} \qquad \forall p \in O^f, \forall q \in Q, \forall l \in O^q, \forall t \in T \tag{20}$$

$$x^t_{slr} = \sum_{p \in P} \sum_{q \in Q} \sum_{c \in C} \sum_{f \in F} x^t_{slrpqcf} \qquad \forall l \in P, \forall r \in R, \forall s \in S^r, \forall t \in T \tag{21}$$

Since this reformulation results in a very large-scale model, decomposition methods are required to solve it. We have considered three decomposition approaches: a cutting-plane method and two Lagrangian relaxations.

The cutting-plane method would start by solving the LP relaxation of the Basic Model. At each iteration, it would generate disaggregated flow variables, along with their corresponding flow conservation equations; this procedure would be carried out only when these variables have a chance to lead to violated forcing constraints. It would then solve the resulting LP relaxation, add violated forcing constraints and iterate until there are no more violated forcing constraints. This method would solve the LP relaxation of the Disaggregated Model; it could be complemented by heuristic techniques and embedded in a branch-and-cut algorithm in order to produce (nearly) optimal solutions to very large-scale problems.

Two Lagrangian relaxations were considered. The first one consists of relaxing the constraints linking consecutive time periods. The resulting Lagrangian subproblem decomposes by time period, and can be solved using the same cutting-plane technique as above. A second Lagrangian relaxation appears more promising, since the resulting Lagrangian subproblem is easier to solve. It consists of relaxing the demand and flow conservation constraints; the resulting subproblem decomposes by node and by time period. It can be shown that both Lagrangian relaxations provide better lower bounds than the LP relaxation.

5. Path Model

Assume we define an *expanded network* consisting of the following types of nodes: couples of the form “supplier-raw material”, couples “plant-intermediate product”, couples “plant-finished product”, and customers. For the first three types of nodes, we will use the notation (o, k) to represent some origin o (supplier or plant) and some commodity k (raw material, intermediate product, or finished product). In this expanded network, there is an arc between two nodes (o, k) and (p, l) if and only if there is an arc between o and p in the original network and $b_{pkl} > 0$ holds. Finally, there is an arc between (p, f) and c if and only if there is an arc between p and c in the original network and $a^t_{cf} > 0$ holds for at least one period t.

Let W denote the set of all paths in this expanded network. Also, we will define the following sets: W_{cf} is the set of all paths delivering finished product f to customer c, and W_{ok} is the set of all paths having (o, k) as starting or intermediate node. We can characterize a path $w \in W$ by its nodes; hence we will use the notation $w = (slrpqcf)$ to denote the path $(s, r) \to (l, q) \to (p, f) \to c$. For such a path, we define *conversion factors* as follows: $b_{wr} = 1$, $b_{wq} = b_{lrq}$, and $b_{wf} = b_{lrq} b_{pqf}$. The transportation cost c_w^t for the path w at time t can then be computed as follows:

$$c_w^t = c_{slr}^t b_{wr} + c_{lpq}^t (b_{wq})^{-1} + c_{pcf}^t (b_{wf})^{-1}.$$

To formulate the path model, we introduce the following path variables: x_w^t is the amount of flow (of raw material) transported on path $w \in W$ at time $t \in T$. The path model can then be written as follows. (Note that the flow conservation constraints are not needed any more.)

$$\min \sum_{t \in T} \sum_{o \in O} c_o^t y_o^t + \sum_{t \in T} \sum_{k \in K} \sum_{o \in O^k} c_{ok}^t v_{ok}^t + \sum_{t \in T} \sum_{w \in W} c_w^t x_w^t \tag{22}$$

subject to

$$\sum_{w \in W_{cf}} (b_{wf})^{-1} x_w^t \leq a_{cf}^t \qquad \forall c \in C, \forall t \in T, \forall f \in F \tag{23}$$

$$\sum_{k \in K^o} \sum_{w \in W_{ok}} (b_{wk})^{-1} x_w^t \leq \overline{u}_o^t y_o^t \qquad \forall o \in O, \forall t \in T \tag{24}$$

$$\sum_{k \in K^o} \sum_{w \in W_{ok}} (b_{wk})^{-1} x_w^t \geq \underline{u}_o^t y_o^t \qquad \forall o \in O, \forall t \in T \tag{25}$$

$$\sum_{w \in W_{ok}} (b_{wk})^{-1} x_w^t \leq \overline{q}_{ok}^t v_{ok}^t \qquad \forall o \in O, \forall k \in K^o, \forall t \in T \tag{26}$$

$$\sum_{w \in W_{ok}} (b_{wk})^{-1} x_w^t \geq \underline{q}_{ok}^t v_{ok}^t \qquad \forall o \in O, \forall k \in K^o, \forall t \in T \tag{27}$$

$$y_o^{t+1} \geq y_o^t \qquad \forall o \in P_0, \forall t \in T \tag{28}$$

$$y_o^{t+1} \leq y_o^t \qquad \forall o \in P_1, \forall t \in T \tag{29}$$

$$y_o^t \in \{0, 1\} \qquad \forall o \in O, \forall t \in T \tag{30}$$

$$v_{ok}^t \in \{0, 1\} \qquad \forall o \in O, \forall t \in T, \forall k \in K \tag{31}$$

$$x_w^t \in \mathbb{R}^+ \qquad \forall w \in W, \forall t \in T \tag{32}$$

The forcing constraints (13), (14), (16), and (18) can then be added to this formulation to strenghthen its LP relaxation. In terms of the path variables, the last three types of forcing constraints can be written as follows:

$$\sum_{w \in W_{cf} \cap W_{pf}} (b_{wf})^{-1} x_w^t \leq a_{cf}^t v_{pf}^t \qquad \forall c \in C, \forall f \in F, \forall p \in O^f, \forall t \in T \tag{33}$$

$$\sum_{w \in W_{cf} \cap W_{pf} \cap W_{lq}} (b_{wq})^{-1} x_w^t \leq (b_{pqf} a_{cf}^t) v_{lq}^t \qquad \forall c \in C, \forall f \in F, \forall p \in O^f, \forall q \in Q, \forall l \in O^q, \forall t \in T \tag{34}$$

$$x_w^t \leq (b_{wf} a_{cf}^t) v_{sr}^t \qquad \forall c \in C, \forall f \in F, \forall p \in O^f, \forall q \in Q, \forall l \in O^q, \forall r \in R, \forall s \in S^r, \forall t \in T, w = (slrpqcf) \tag{35}$$

It can be shown that the LP relaxation bounds provided by the Path Model and by the Disaggregated Model are equal. To solve the LP relaxation of the Path Model, a column generation approach can be used. This technique starts by generating a small subset of all paths, and at each iteration, solves the resulting restricted LP relaxation. New (non-basic) path variables with negative reduced costs are then generated by solving the pricing subproblem. We still need to investigate the structure of this pricing subproblem to determine whether or not it can be solved efficiently.

6. Conclusion

In this report, we have presented the work carried out by the team of researchers and students who studied the strategic planning problem of Kruger Products during the First Montreal IPSW. The team proposed three formulations for this problem: the Basic, Disaggregated, and Path Models. The first one can be solved by a state-of-the-art MIP solver, but its LP relaxation is notoriously weak when fixed charges are high and capacities are tight. The two other models provide better LP relaxations, but require decomposition methods if one is to solve large-scale instances. The team has outlined three approaches for the Disaggregated Model: a cutting-plane method and two Lagrangian relaxations; it has also outlined a column generation approach for the Path Model.

Before implementing these decomposition approaches, it is first necessary to test the Basic Model, both on randomly generated instances and on real data. Then the two other models could be implemented and tested on small instances, without using decomposition, in order to determine to what extent their LP relaxation value improves the bound obtained by solving the LP relaxation of the Basic Model. Finally, decomposition approaches could be implemented and tested on large-scale instances.

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