The Flat Iron Device for Measuring Changes in Conductivity

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Abstract. This note proposes an alternative to both the 'transient hot-wire method' and the 'transient hot-strip method'. A mathematical model is developed and an analytic solution is provided. The 'flat iron device' may have advantages for rapidly measuring changes in conductivity, as can occur in food products.

1. Introduction

It has been argued by Gustafsson et al (see eg. Gustafsson et al 1979, 1984, 1991) that the transient hot strip method is superior to the transient hot wire method (see eg. McLaughlin and Pittman (1971)) since the hot wire method is restricted to fluids or other substances than can envelop the wire. They point out that the hot strip method can now deal with any solid with low electrical conductivity; this is achieved by using the metal strip both as a continuous plane heat source and as a sensor of the temperature increase in the strip itself.

The power will remain very nearly constant if a constant current is supplied to the metal strip. Thus it is possible to obtain the thermal properties of the material surrounding the heat source by monitoring the voltage increase over a short period of time. The temperature increase causes an increase in the electrical resistance of the metal strip with a consequent voltage increase.

For this device to work well it is necessary to press two slabs of material against the metal strip so that the strip approximates a plane heat source. If, however, there is only one specimen material this device becomes less attractive and probably not applicable. Yet the need to measure conductivity (or more precisely, changes in conductivity) is not uncommon. For instance, in the Food Industry it would be extremely useful to have a device which could rapidly measure changes in the thermo-physical properties of a single specimen, for example a piece of fish or a beefburger.

The purpose of this note is simply to put forward the idea of a flat iron device, to build a mathematical model and develop an analytic expression for the temperature at its surface, thereby suggesting its feasibility.

2. The flat iron device

The flat iron device, as the name suggests, is a rectangular block of iron. Its 'flat' surface contains many parallel shallow grooves in which is embedded an insulated cable and a resistor. A constant current is passed through each resistor which varies linearly with the temperature: the temperature at the surface is then determined by the average voltage drop. A cross-section is displayed in diagram 1.

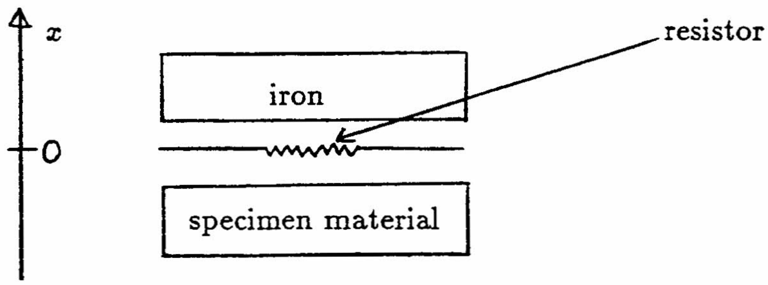


diagram 1

3. The mathematical model

Let T_1 and T_2 be, respectively, the temperatures in the iron and the specimen, and let T_A denote the ambient temperature. On the assumption of many parallel heating resistors and that the interest is only in the temperature near to the heating elements a one dimensional model will suffice. Let x point in the outward direction from the heating element as shown in diagram 1.

The model can be expressed as two coupled diffusion equations as follows:

$$\rho_1 c_1 \frac{\partial T_1}{\partial t} = k_1 \frac{\partial^2 T_1}{\partial x^2}$$

$$\lim_{x \to \infty} T_1(x,t) = T_A, \quad T_1(x,0) = T_A$$

$$\rho_2 c_2 \frac{\partial T_2}{\partial t} = k_2 \frac{\partial^2 T2}{\partial x^2}$$

$$\lim_{x \to -\infty} T_2(x,t) = T_A, \quad T_2(x,0) = T_A$$

together with the coupling boundary conditions

$$T_1(0,t) = T_2(0,t)$$

$$-k_1 \frac{\partial T_1}{\partial x} + k_2 \frac{\partial T_2}{\partial x} = \alpha(1 + \beta T_1), \quad \alpha > 0, \text{ at } x = 0 \text{ for all } t,$$

where ρ_i, c_i, and $k_i (i = 1,2)$ are, respectively, the density, specific heat and thermal diffusivity of the iron and the specimen, and α and β are known constants depending upon both the constant current and how the heater wire changes with temperature.

By introducing the new dependent variables

$$T_1' = T_1 - T_A, \qquad T_2' = T_2 - T_A$$

the above initial-boundary problem reduces to

$$\frac{\partial T_1}{\partial t} = K_1 \frac{\partial^2 T_1}{\partial x^2} \tag{1a}$$

$$\lim_{x \to \infty} T_1(x,t) = 0, \qquad T_1(x,0) = 0 \tag{1b,c}$$

$$\frac{\partial T_2}{\partial t} = K_2 \frac{\partial^2 T_2}{\partial x^2} \tag{2a}$$

$$\lim_{x \to -\infty} T_2(x,t) = 0, \qquad T_2(x,0) = 0 \tag{2b,c}$$

where $K_1 = k_1/(\rho_1 c_1)$ and $K_2 = k_2/(\rho_2 c_2)$, with the coupling conditions

$$T_1(0,t) = T_2(0,t) \tag{3a}$$

$$-k_1 \frac{\partial T_1}{\partial x} + k_2 \frac{\partial T_2}{\partial x} = \alpha(1 + \beta T_A + \beta T_1), \ \alpha > 0, \text{ at } x = 0 \text{ for all } t. \tag{3b}$$

Note that the dashes have been dropped for reasons of clarity.

4. Analytic solution

Define the Laplace transforms with respect to t

$$\overline{T}_i(x,p) = \int_0^\infty e^{-pt} T_i(x,t)\, dt \qquad i = 1,2.$$

Equation (1a) (with (1b,c)) reduces to

$$p\overline{T}_1(x,p) = K_1 \frac{\partial^2 \overline{T}_1}{\partial x^2}(x,p)$$

yielding the general solution

$$\overline{T}_1(x,p) = A_1(p)e^{\sqrt{q_1}x} + B_1(p)e^{-\sqrt{q_1}x}, \ q_1 = p/K_1,$$

where $A_1(p)$ and $B_1(p)$ are arbitrary functions of p.

Similarly, equation (2a) (with (2b, c)) yields the solution

$$\overline{T}_2(x,p) = A_2(p)e^{\sqrt{q_2}x} + B_2(p)e^{-\sqrt{q_2}x}, \; q_2 = p/K_2,$$

where $A_2(p)$ and $B_2(p)$ are arbitrary functions of p.

Conditions (1b) and (2b) imply $A_1(p) = B_2(p) = 0$ giving

$$\begin{aligned}\overline{T}_1(x,p) &= B_1(p)e^{-\sqrt{q_1}x} \\ \overline{T}_2(x,p) &= A_2(p)e^{\sqrt{q_2}x}\end{aligned}.$$

Further, from (3a) we deduce that $B_1(p) = A_2(p) = A(p)$, say, resulting in

$$\overline{T}_1(x,p) = A(p)e^{-\sqrt{q_1}x} \tag{4}$$

and

$$\overline{T}_2(x,p) = A(p)e^{\sqrt{q_2}x}. \tag{5}$$

Now taking Laplace transforms with respect to time in (3b) results in

$$-k_1 \frac{\partial \overline{T}_1}{\partial x}(x,p) + k_2 \frac{\partial \overline{T}_2}{\partial x}(x,p) = \frac{\alpha(1+\beta T_A)}{p} + \alpha\beta\overline{T}_1(x,p)$$

giving, upon using (4) and (5),

$$k_1\sqrt{q_1}A(p) + k_2\sqrt{q_2}A(p) = \frac{\alpha(1+\beta T_A)}{p} + \alpha\beta A(p)$$

at $x = 0$.

Hence

$$A(p) = \frac{\alpha(1+\beta T_A)}{p\{(k_1/\sqrt{K_1} + k_2/\sqrt{K_2})\sqrt{p} - \alpha 3\}}. \tag{6}$$

Thus, substituting (6) into (4) gives

$$\overline{T}_1(x,p) = \frac{\alpha_0 \, e^{-\sqrt{q_1}x}}{\alpha_1(p(\sqrt{p}+a))} \tag{7}$$

where

$$\begin{aligned}\alpha_0 &= \alpha(1+\beta T_A) \\ \alpha_1 &= \frac{k_1}{\sqrt{K_1}} + \frac{k_2}{\sqrt{K_2}} \\ a &= -\alpha\beta/\alpha_1.\end{aligned}$$

Taking inverse Laplace transforms of (7) yields

$$T_1(x,t) = \frac{\alpha_0}{\alpha\beta}\left\{e^{-ax/\sqrt{K_1}}e^{a^2t}\operatorname{erfc}\left(a\sqrt{t} + \frac{x}{2\sqrt{K_1 t}}\right) - \operatorname{erfc}\left(\frac{x}{2\sqrt{K_1 t}}\right)\right\}$$

(see Abramowitz and Stegun, page 1027).

Hence

$$T_1(0,t) = \frac{\alpha_0}{\alpha\beta}\left\{e^{a^2t}\operatorname{erfc}(a\sqrt{t}) - 1\right\}. \tag{8}$$

5. Graphical presentation

In this section the temperature at the surface of a flat iron device which is made of aluminium and residing against a meat product is displayed graphically. The densities, specific heats and thermal diffusivities are displayed in Table 1. The graphs were obtained by plotting the expression (8) against time. The results are not surprising showing a steady (and eventually unbounded) increase in the temperature. A meat product which had 'gone off' would display a different temperature profile as a result of the changed thermal properties and so could be relatively rapidly identified.

	ρ	c	k
Aluminium	$2700 Kg/m^3$	$0.9 KJ/Kg^\circ C$	$240 W/m^\circ C$
Food (meat)	$1050 Kg/m^3$	$3.5 KJ/Kg^\circ C$	$0.5 W/m^\circ C$

Table 1

References

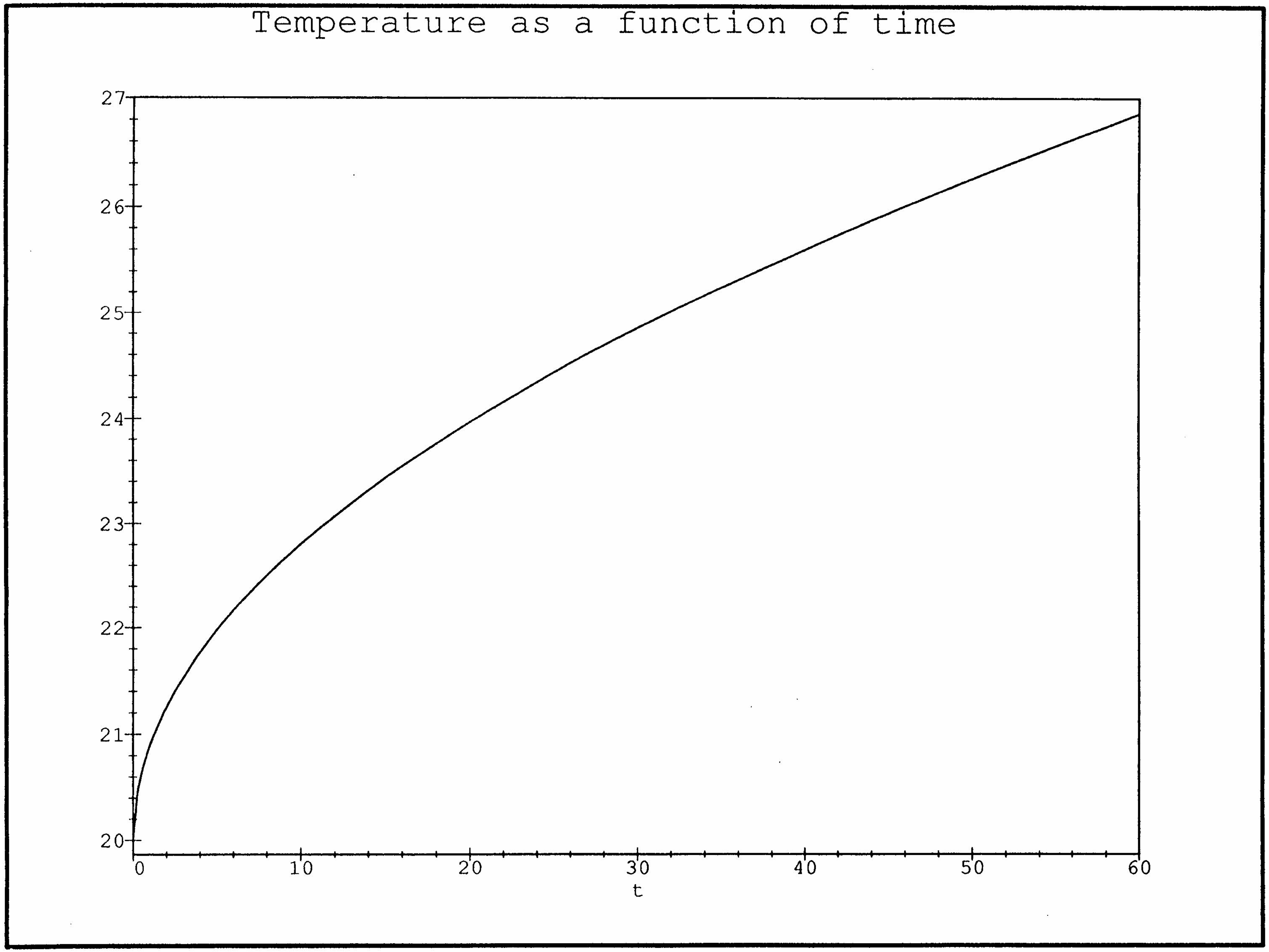
Abramowitz and Stegun, I (1964) Handbook of Mathematical Functions.

Gustafsson, S E, Karawacki, E and Khan, M N (1979) Transient hot-strip method for simultaneously measuring thermal conductivity and thermal diffusivity of solids and fluids. J. Phys. D : Appl Phys. **12**, 1411–1421.

Gustafsson, S E and Karawacki, E (1984) Circuit design for transient measurements of electrical properties of thin metal films and thermal properties of insulating solids or liquids. Rev. Sci. Instrum. **55**, No.4, 610–613.

Gustafsson, S E (1991) Transient plane source techniques for thermal conductivity and thermal diffusivity measurements of solid materials. Rev. Sci. Instrum. **62**, No.3, 797–804.

McLaughlin, E and Pittman, J F T (1971) Phil. Trans. Roy. Soc. A, **270**, 557–579.


Temperature as a function of time
27
26
25
24
23
22
21
20
0
10
20
30
40
50
60
t