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Comparable aggregated indicators of QoS in the telecoms market

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REPORT ON THE PROBLEM

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Executive Summary

The Polish telecommunication company TP SA provides diversified services for alternative operators (AO). The quality of these services is measured periodically using the so-called Key Performance Indicators (KPI).

There are more than 500 alternative operators and 63 different KPIs that measure the quality of services. The Office of Electronic Communications (UKE) receives periodic reports for each operator with specific data which can be arranged in one table. This table has a lot of cells that are not filled and its structure may differ in time, which poses the problem of how to compare alternative operators and indicate eventual discrimination. Proposed was the concept of defining discrimination. This approach is based on some additional parameters, which were determined, however, they have not been collected so far.

The main challenge was to design aggregated indicators measuring the quality of services rendered by the wholesale telecommunication operator to alternative operators. The indicators must be comparable, i.e. they should indicate whether some AOs are favoured or discriminated.

As a result, two different methods of computing aggregated indicators were proposed. The first one, Principal Component Analysis, is based on the reduction of data dimension, which facilitates further analysis. It shows whether some AOs are treated in a different way, when compared with others. This may indicate discrimination.

The second method aggregates all values of KPIs and assigns a real number to each alternative operator. Subsequently, rankings of AOs treatment can be set up. This method enables detection of a discriminated operator and was tested in simulations.

Contents

1 Introduction

1.1 Background

(1.1.1) For historical reasons, the Polish telecommunication company TP SA has always had a dominant position in the Polish telecoms market. This monopolistic situation hindered development of the telecommunication market in Poland, which resulted in relatively high prices and a low quality of services.

(1.1.2) The regulatory activities for the telecom market in Poland are provided by the Office of Electronic Communications (UKE). After many complaints and reports about monopolistic practices of TP SA, UKE imposed in 2009 an agreement between UKE, alternative operators (AOs) and TP SA. The TP-Wholesaler (TPW), operating within the structure of TP SA, should secure an equal access and the same level of wholesale service to all alternative operators (independent ones as well as those related to TP SA).

(1.1.3) In order to check if the agreement is fulfilled properly, a set of quantitative indicators (Key Performance Indicators – KPIs) was defined to measure the quality/level of various aspects of provided services [1]. In particular, this shall enable UKE to control whether some of AOs are favoured or discriminated by the TPW.

(1.1.4) UKE receives values of KPIs (over 60) for every AO (about 500) for each defined time period. Obtained data can be collected into one table. The problem arises due to the fact that some cells in the table may be empty (no data available). Additionally, dimension and structure of the table may vary between periods. Thus, there is a problem of how to interpret data properly.

1.2 Problem description

(1.2.1) *Specification of input data*

KPIs are of various types (usually integers and percentages) and ranges.

AOs choose different subsets of services offered by the TPW. Thus, some KPIs may not be defined for a given AO.

Some KPIs may be indefinite for certain reporting periods, although services related to these KPIs are provided (e.g. because of the lack of data).

The set of KPIs for individual AOs may vary for various reporting periods (e.g. because some services are added or withdrawn from the offer of TPW).

The set of indefinite KPIs for a given AO may be different for various reporting periods (e.g. because the AO starts or stops using some services).

(1.2.2) *Main challenge*

The main aim was a construction of aggregated indicators that measure the level of service rendered by TPW to AOs on the basis of the table with KPIs. Proposed indicators must be comparable, which means that they should indicate whether some AOs are favoured or discriminated.

(1.2.3) *Secondary challenge*

Aggregated indicators should allow comparing between several reporting periods.

1.3 Problem of discrimination

(1.3.1) The following possibilities of eventual discrimination are considered:

1. Having data for operators **A** and **B**, one can deduce that operator **A** is discriminated when compared with operator **B**.

2. Having data for one operator, it can be stated whether it is discriminated or not.

3. With reference to the real number, which is an output variable characteristic for an individual operator, one can decide which operator is discriminated.

The third option seems to be the most promising when dealing with the presented problem.

(1.3.2) *Proposal for evaluation of discrimination*

The current set of KPIs should be extended and divided into the following groups:

1. *Incumbent's preference*

This group of KPIs is characterised by the aggregated parameter $\mathrm{P1}$. The group consists of the 63 given KPIs. Parameter $\mathrm{P1}$ measures the level of service rendered by the Incumbent to an alternative operator.

The aggregated parameter $\mathrm{P1}$ calculated for particular AO shows whether this AO is favoured or discriminated in comparison to another AO.

2. *Operator's quality*

This group of KPIs is characterised by the aggregated parameter $\mathrm{P2}$. KPIs for this group are not defined at this moment. These KPIs are additional to the existing 63 KPIs and can be collected from AOs or open informational resources. They characterise the quality of work of each AO.

Definition of discrimination

Operator **A** is discriminated in the market if operator **B** is such that: $\mathrm{P1(A)} < \mathrm{P1(B)}$ – Incumbent renders lower service to operator **A** than to operator **B** and $\mathrm{P2(A)} > \mathrm{P2(B)}$ – operator **A** provides higher quality service than operator **B**.

2 Proposed methods for evaluation of aggregated indicators

(2.0.1) Proposed are two methods for evaluation of aggregated indicators and identification of discriminated alternative operators.

Thanks to the result of computations by means of the proposed methods, we have obtained an ordered list of operators.

The existence of some linear ordering in the set of operators does not reveal whether operators are discriminated.

Differences between (aggregated) KPIs may result from simple statistical fluctuations or inaccuracies in measurements of the KPIs. Answering the question if there is discrimination or not requires tools which allow distinguishing between the differences in values of (aggregated) KPIs coming from random fluctuations from those indicating unfair treatment.

Methods of aggregation of the KPIs generate operators which are suspected to be discriminated if the values of aggregated KPIs differ significantly from the mean value. These operators are called outliers.

Having the set of outliers one should answer the question if AO is discriminated or not by formulating, on a given level of confidence, a statistical hypothesis concerning the extent to which being an outlier can be a random statistical fluctuation.

2.1 Principal Component Analysis (PCA)

Theoretical background

(2.1.1) Theoretical introduction for the PCA described below is based on [2], [3] and [4].

Theoretically, the PCA is the optimal linear scheme, in terms of a least mean square error, for compressing a set of high dimensional vectors into a set represented by vectors of a relatively lower dimension.

This is a *non-parametric* analysis and the answer is both *unique* and no assumptions about data probability distributions are made. No ex-ante assessment is required and the first principal component has the highest variance among all components. However, one should be aware of the fact that some important information can be lost when using the PCA method.

(2.1.2) Let $X=(x_{ij})$ be a matrix of a dimension of $n\times p$ where row *i* represents the *i*-th observation (alternative operator) as a vector $x_i'=(x_{i1},\cdots,x_{ip})$ [1]. The PCA approach consists of a set of projections (with a set of basis vectors $a_1,\cdots,a_p\in\Re$) of the multivariate data, which are mutually uncorrelated and ordered in variance. The following properties of projections are postulated, viz.:

1. lengths of vectors a_j are equal to 1,
2. vector a_1 defines such a direction that projections $a_1'x_1,\cdots,a_1'x_n$ have the largest variance, vector a_2 defines an orthogonal vector that explains the remaining portion of the variance etc. Vector a_j is called also the *j*-th *loading* and is calculated as an eigenvector for the covariance/variance matrix of a sample[2] S corresponding to the *j*-th largest eigenvalue λ_j of this matrix. The eigenvalue λ_j is equal to a variance within the sample $a_1'x_1,\cdots,a_1'x_n$. Value $y_j=a_j'x$ is the *j*-th principal component of vector x .

Standarization of KPIs

(2.1.3) Due to their diversification KPIs must be standardized. In the PCA method all KPIs were standardized to a zero mean and a standard deviation of 1. Thus, the *j*-th standardized attribute for the *i*-th operator is equal to:

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[1] x' is a transposition of vector x.

[2] input data in our case.

$$KPI_s^{ij} = \frac{KPI^{ij} - \overline{KPI^j}}{\sigma_{KPI^j}}, \quad (1)$$

where $\overline{KPI^j}$ and σ_{KPI^j} are mean and standard deviation of the *j*-th attribute respectively for all non-missing values (*null* values were not taken into account). After the standardization, all missing values are substituted with zeros. This strategy may not seem to be prudent at first. However, it is correct for both statistical properties of the PCA, as well as the order of standardization and data imputation. Indeed, thanks to these operations, standard deviations of attributes with a relatively higher number of missing values decrease, which results in lower importance of respective KPI while using the PCA[3]. It should be noted that KPIs with zero deviation must be excluded from the analysis, because they do not provide any information about discrimination.

2.2 Function method

(2.2.0) Notation

Let:

I - set of KPIs indexes,

o − an alternative operator,

$D(o) \subseteq I$ - set of definite attributes for an operator o,

$D'(o) \subseteq D(o)$ - set of definite comparable attributes for an operator o,

$x \in D'(o) \Leftrightarrow x \in D(o) \wedge \exists o' \neq o \wedge x \in D(o')$,

i – an index of KPI,

$r_i(o)$ – position of an operator o among the values of the *i*-th KPI sorted non-decreasingly.

Standarization of KPIs

(2.2.1) In the proposed method, the set of KPIs is divided into the following four groups:

A – KPIs with reference value equal to 1,

B – KPIs which are average numbers of specific events out of one hundred events and their reference value is equal to 0,

C – KPIs with values expressed in hours,

D – remaining KPIs with reference value equal to 0.

(2.2.2) During the proposed process of standarization:

- Values of KPIs from the *A* group are not changed.

[3] Note that this method "prefers" attributes exhibiting higher dispersion.

- Values of KPIs from the *B* group are standardized with use of the following formula:

$$val(o,i) = 1 - \frac{val'(o,i)}{100}, \qquad (2)$$

where $val'(o,i)$ is value of the *i*-th KPI for the operator o before standardization and $val(o,i)$ is its value afterwards.

- KPIs from the *C* group are standardized as follows:

$$val(o,i) = \frac{val'(o,i)}{\max(val'(o,i))}, \qquad (3)$$

where the maximum value is taken over all AOs, which have the *i*-th KPI.

- KPIs from the *D* group are standardized with formula given below:

$$val(o,i) = 1 - val'(o,i). \qquad (4)$$

Aggregated indicator $F(o)$

(2.2.3) The first proposition for the aggregated indicator is given by the following function:

$$F(o) = \frac{1}{|D(o)|} \sum_{i \in D(o)} (val(o,i) - \mu_i), \qquad (5)$$

with the mean value μ_i for the *i*-th KPI.

It assigns a real number to each AO, which is the measure of potential discrimination. The negative value of the indicator can be interpreted as discrimination in comparison to the mean value of service quality measured by KPIs.

The indicator is normalized with respect to the number of KPIs that is definite for a given operator to ensure that its value for AOs with different number of definite KPIs is comparable.

Aggregated indicator $Ord(o)$

(2.2.4) The second proposal for the aggregated indicator is given by the following function:

$$Ord(o) = \frac{1}{|D'(o)|} \sum_{i \in D'(o)} \frac{r(o,i)}{O(i) - 1} \qquad (6)$$

Consider a single indicator KPI_i. Suppose that the operator *o* provides a service which is measured by this KPI_i and $O(i)$ is the mean value of this KPI_i for all considered operators. The ranking of operators due to given KPI_i is obtained by sorting in descending order with respect to the KPI_i values for all operators which provide the same service (numbering from 0). The $r(o,i)$ value is the number on the sorted list and the ratio $\frac{r(o,i)}{O(i)-1}$ is the fraction of the number of operators with greater (lower) KPI_i value to the number of all operators for which the given KPI_i is defined. Information concerning the group of operators that was treated better (or worse) is obtained after computing the average value, c.f. Eq. (6).

(2.2.5) The higher value of $Ord(o)$, the greater discrimination of the operator *o*.

If KPIs values are equal for all operators, then $Ord(o) = 0$ for every operator.

If all operators are treated in a similar way, $Ord(o)$ should be relatively small.

3 Implementation

(3.0.1) Implementation of the proposed methods is described in this section.

Computations are based on the real values of KPIs from the periodic report delivered by a representative of UKE.

3.1 Implementation of the PCA

(3.1.1) The outcome of the analysis takes the form of principal components. Presented in Fig. 1, the two first principal components have the highest contribution to the variance of the data (over 70%, c.f. Fig. 2). Values of the three major principal components for the *i*-th operator can be calculated in a following way:

$$PC_i^I = -0{,}91\,KPI_{i;18} - 0{,}32\,KPI_{i;11} - 0{,}15\,KPI_{i;45} + 0{,}1\,KPI_{i;55} + R_i^I \tag{7}$$

$$PC_i^{II} = -0{,}997\,KPI_{i;55} + R_i^{II} \tag{8}$$

$$PC_i^{III} = -0{,}997\,KPI_{i;11} + R_i^{III} \tag{9}$$

where:

$KPI_{i;18}$ - punctuality of invoicing,

$KPI_{i;11}$ - punctuality of replying to a ROI query,

$KPI_{i;45}$ - indicator for correctness of information given by the Incumbent through ISI, see [1],

$KPI_{i;55}$ - average time of executing orders for Internet services.

These particular KPIs and respective coefficients were chosen by the PCA.

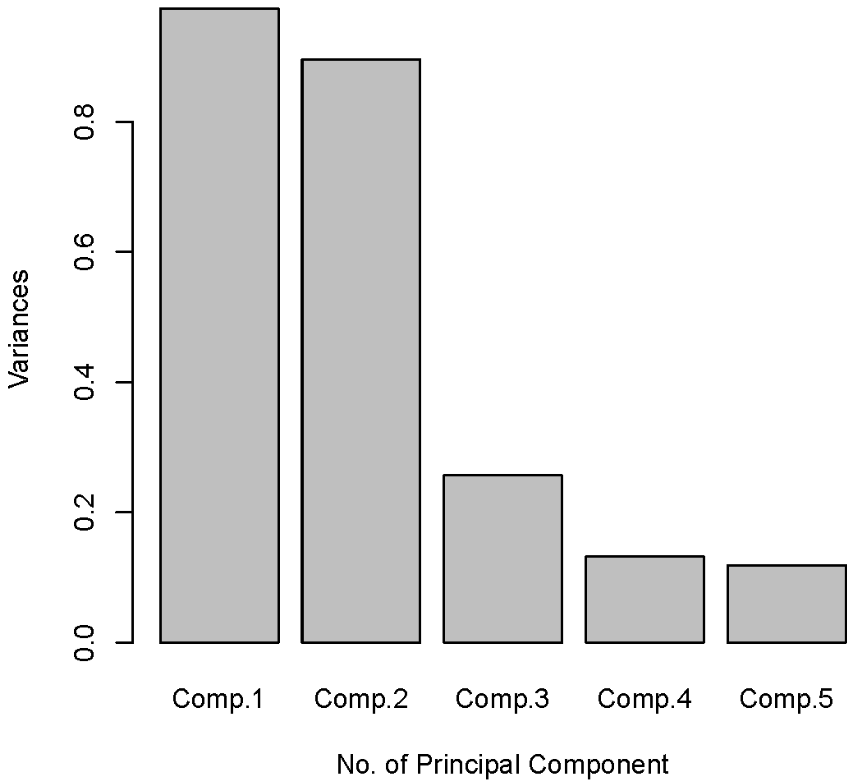

Figure 1. Variance contributions for each principal component.

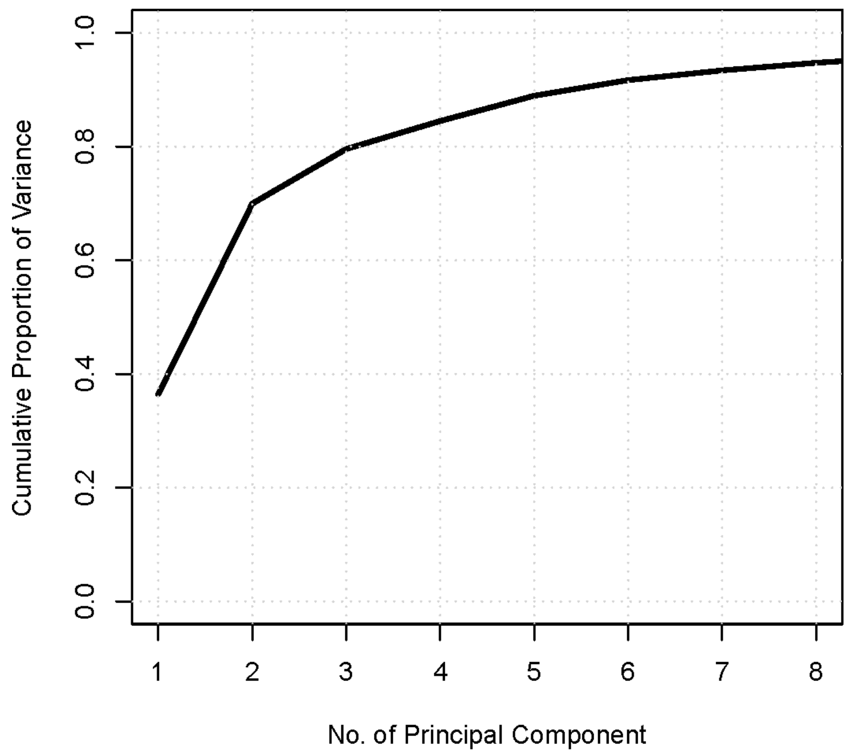

Figure 2. Cumulative proportion of variance.

R quantities are parts of Eq. (7-9) which have a relatively small impact on the value of PCAs[4]:

$$R_i^I = \sum_{j \in KPI_s \setminus \{18,11,45,55\}} \alpha_j KPI_{i;j} , \tag{10}$$

$$R_i^{II} = \sum_{j \in KPI_s \setminus \{55\}} \alpha_j KPI_{i;j} , \tag{11}$$

$$R_i^{III} = \sum_{j \in KPI_s \setminus \{11\}} \alpha_j KPI_{i;j} , \tag{12}$$

where α_j is the coefficient of the linear combination of principal components.

(3.1.2) The analysis identifies outliers in the space of principal components. Nonetheless, it cannot be ascertained whether outliers are discriminated or favoured by the Incumbent. For this reason, signs of coefficients in Eq. (7-9) have to be assessed. Outliers in a dimension of PC^I show the relatively highest value of the component. Coefficients for this component are negative if higher values of attributes denote a better quality of services provided by the Incumbent. Therefore, a higher value of PC^I denotes a higher level of discrimination if compared to other operators.

The operator AO324 is the most discriminated according to the criterion of PC^I, the linear combination of KPI_{18}, KPI_{11}, KPI_{45} and KPI_{55}, what is shown in Fig. 3. The second most discriminated operator is AO147.

According to PC^{II}, outliers are significantly below the average. The coefficient of the highest importance represented by KPI_5[5] is positive. An outlier of a negative value for this component means discrimination for this operator. It is similar in the case of PC^{III}. The only outlier in this dimension exhibits a value significantly below the average of the principal component.

An analysis across PC^{II} driven by KPI_5 allows detecting more discriminated operators. These operators are (in the ascending order):

AO9,
AO364,
AO355,
AO530,
AO433.

An additional analysis across PC^{III} confirms that operator AO147 is the most discriminated.

[4] R^I explains only 3.7% of a variance of PC^I, R^{II} merely 0.3% and R^{III} a slightly higher level of 19% (however, on the other hand, the marginal contribution of PC^{III} to the variance is low and accounts for merely about 10%, c.f. Fig. 2).

[5] The higher the value of this KPI, the better is the quality of service provided by the Incumbent.

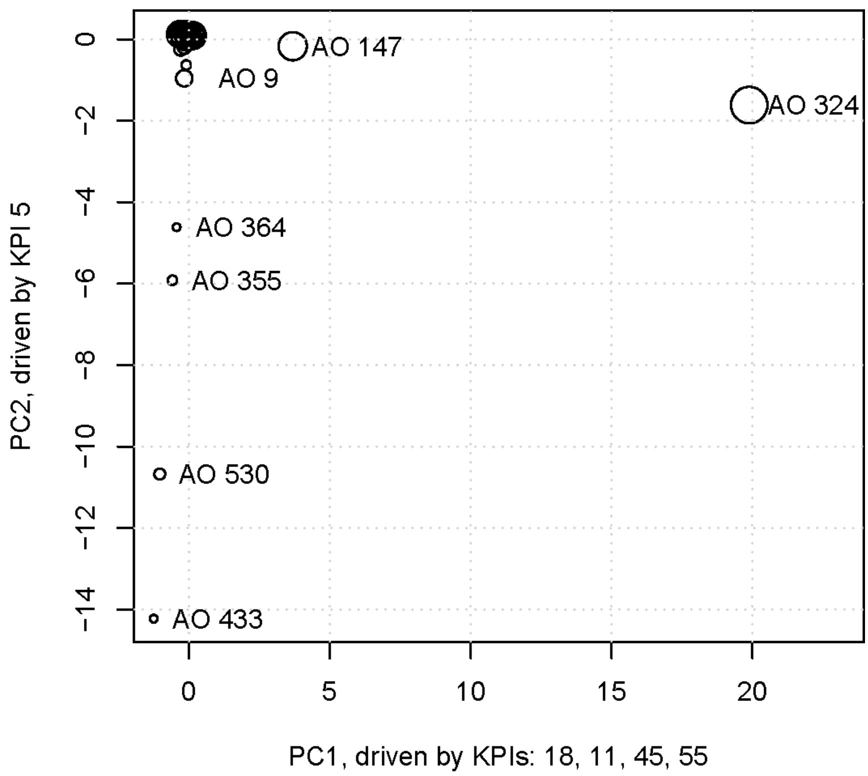

Figure 3. 2-D projection in the (PC^I, PC^{II}) plane.

(3.1.3) In order to achieve one-dimensional indicator, the combination of the first two principal components was created. It explains 70% of the variance of the input data:

$$D_i = \sum_{j \in KPI_s} a_{j;1} \cdot \frac{KPI^{ij} - \overline{KPI^j}}{\sigma_{KPI^j}} + \omega_5 KPI_{i;j}, \tag{13}$$

where $a \in \mathbb{R}^{KPI_s}$ is a vector of loadings which is a solution to the following program:

$$\max_{\|a\|=1} \{Var(a'x)\}, \tag{14}$$

and ω_5 is a trade-off parameter determining a dominance boundary between two criteria. Value of this parameter was chosen arbitrarily and should be set after consultations with experts. For the given data an optimal value of this parameter is equal to 1.5.

Fig. 4 shows values of the indicator (13) for $\omega_5 = 1.5$. The following list of the most discriminated operators was obtained with the cut-off value of 2:

AO324,
AO433,
AO530,
AO355,
AO364,
AO147.

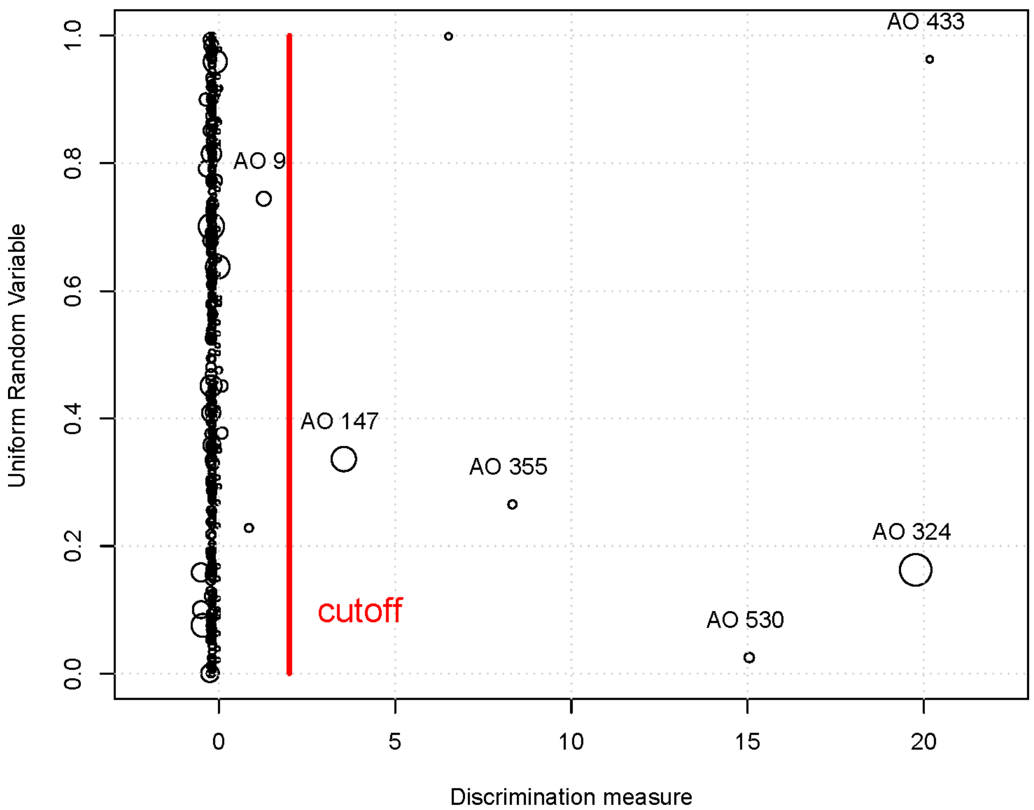

Figure 4. One-dimensional discrimination indicator.

3.2 Implementation of function method

(3.2.1) Experiments were carried out on the real data set. The discriminated "big" operator was identified correctly by at least one of the aggregated indicators (F : 1 mistake, Ord : 3 mistakes out of 11 simulations) c.f. Table 1. Both indicators show partially overlapping sets of discriminated operators. The Incumbent can hide important discriminated operators by lowering intentionally the KPIs values for smaller operators.

The order of discrimination amongst "big" operators is shown in Table 2.

Discriminated operators are these which have high value in both rankings.

Table 1. Results with use of the Ord and F indicators. Only top 14 operators are shown.

operator	Ord indicator	operator	F indicator	overlap
433	0.5	433	0.3374	1
364	0.4960	376	0.2051	1
296	0.4765	530	0.1265	1
324	0.4078	364	0.1095	2
9	0.3834	296	0.1068	3
114	0.3684	513	0.1035	3
147	0.3361	355	0.0934	3
143	0.3333	143	0.0459	4
355	0.3315	9	0.0262	6
534	0.3314	114	0.0253	7
342	0.3302	472	0.0195	7
345	0.3289	245	0.0132	7
513	0.3034	528	0.0131	8
270	0.28	270	0.0046	9

Table 2. Values of indicators F and Ord for “big” operators.

operator	Ord indicator	operator	F indicator	overlap
324	0.4078	513	0.1035	0
147	0.3361	245	0.0132	0
534	0.3314	270	0.0046	0
513	0.3034	498	0.0037	1
270	0.2842	534	0.0034	3
273	0.2809	273	-0.0026	4
282	0.2676	324	-0.0094	5
498	0.2470	282	-0.0097	7
245	0.2255	147	-0.0149	9
235	0.1614	235	-0.0191	10

Calculations on different real data subsets

(3.2.2) One of the performed tests was a calculation of F indicator values, based on different subsets of real data provided by the UKE.

The matrix of data provided by the UKE was modified in the following way:
- values x of KPIs for which the reference value is 0 were replaced by $1-x$;
- values of one KPI (number 55) expressed in hours were divided by maximum value (102h) to obtain a number in a range between 0 and 1;
- columns representing KPIs were sorted decreasingly with respect to the number of operators for which the given KPI has a definite value;
- rows representing operators were sorted decreasingly with respect to the number of definite KPIs for a given alternative operator.

As a result, definite values are grouped in upper-left corner, while rows and columns less populated with data are suppressed to the right and lower part of the table.

(3.2.3) Four submatrices were chosen from the table prepared in the way described above:

1. The table of all operators and all KPIs, dimension 562 x 62 (almost empty). After removal of empty rows and empty columns from the data delivered by the UKE, the table with the dimension 425 x 48 is obtained. Since the empty rows and columns do not carry any information, the reduced table is analyzed further.

2. The table of 32 largest operators and KPIs with at least 5 fields filled, dimension 32 x 31 (partially filled).

3. The table of 14 largest operators and KPIs with at least 8 fields filled, dimension 14 x 15 (half-filled).

4. The table of 14 largest operators and KPIs with at least 10 fields filled, dimension 14 x 12 (almost full).

(3.2.4) In each case only 14 largest operators, which appear in the last set, were taken into consideration. The results of how they are treated with respect to each data set are displayed in the Table 3.

Fig. 5 shows the position of AO calculated on the basis of F values. The first three operators remain on the top of the list, while there is a mix in the middle, as different data subsets are considered. It concerns such AOs for which values of F are very close to each other, which can be seen in the next figure, where these values are plotted.

Values of F are displayed for each data set in Fig. 6. Operator No 513 dissents from the others and is constantly on the top of the list. The values of F for operators which are in the middle of the list are very similar.

Table 3. Results for the chosen operators according to different dimensions of input matrix.

Operator number	Number of definite KPIs	Value of F	Place in the ranking	Value of F	Place in the ranking	Value of F	Place in the ranking	Value of F	Place in the ranking
513	15	-0,1036	1	-0,1041	1	-0,0455	1	-0,0797	1
245	20	-0,0133	2	-0,0123	3	-0,0161	2	-0,0400	2
528	11	-0,0131	3	-0,0132	2	-0,0137	3	-0,0137	3
270	25	-0,0046	4	-0,0057	4	-0,0041	5	0,0112	9
532	13	-0,0031	7	-0,0017	6	0,0032	7	0,0032	5
470	14	0,0048	9	0,0007	7	0,0019	6	0,0019	4
324	45	0,0095	10	0,0116	11	-0,0047	4	0,0043	6
273	15	0,0027	8	0,0026	9	0,0048	10	0,0048	7
498	24	-0,0038	5	0,0012	8	0,0205	13	0,0058	8
534	24	-0,0035	6	-0,0048	5	0,0042	9	0,0230	14
282	29	0,0098	11	0,0104	10	0,0041	8	0,0188	10
147	27	0,0150	12	0,0157	12	0,0128	11	0,0225	13
235	17	0,0192	13	0,0189	13	0,0191	12	0,0191	11
387	10	0,0206	14	0,0208	14	0,0220	14	0,0220	12
		1: 425 x 48		2: 32 x 31		3: 14 x 15		4: 14 x 12	

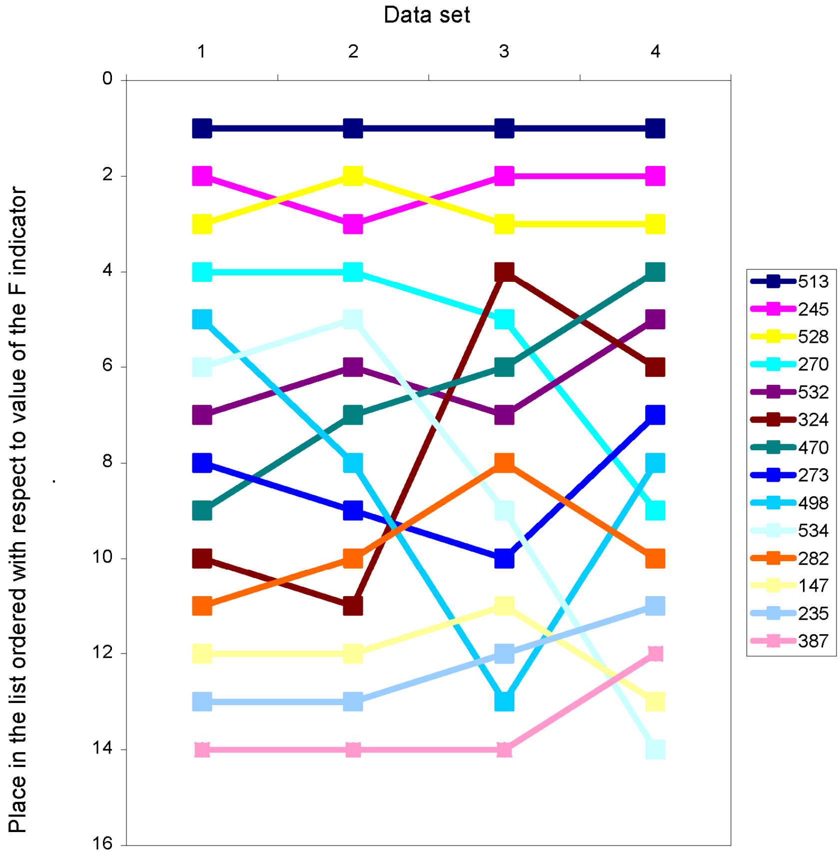

Figure 5. Ranking of AO (only 14 positions are shown) with respect to different data subsets calculated with the use of F indicator.

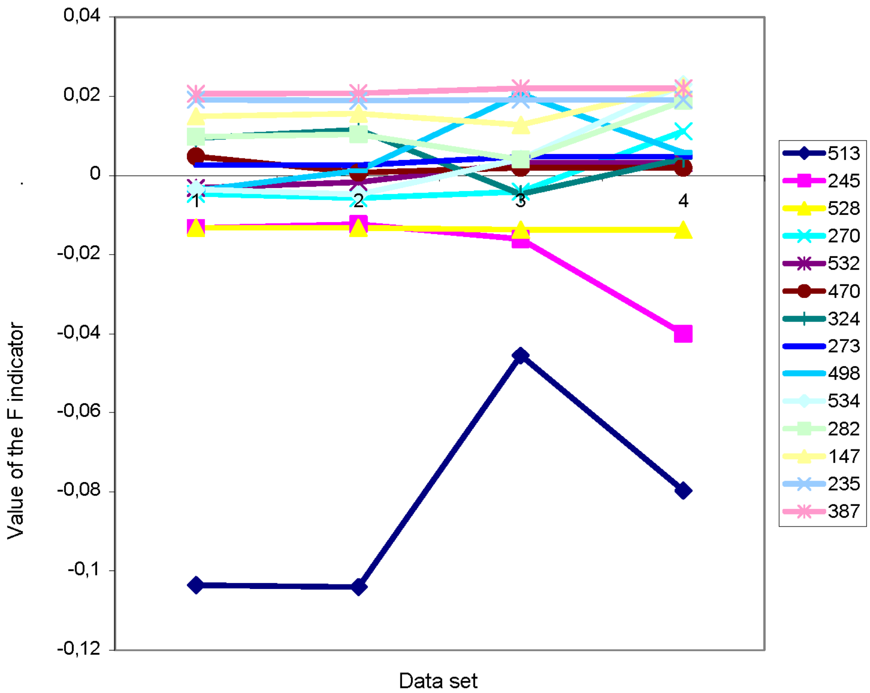

Figure 6. Values of F indicator for different AOs and four data subsets.

(3.2.5) The values of the F indicator do not vary significantly for different data subsets. It means that this method for calculating aggregated KPI is not sensitive in case of the lack of data. It should be noticed that the first data set is a table with about 95% of empty fields and the last one (data set number 4) is almost full with only few fields without data.

Having the table sorted, one can truncate it from the right and from the bottom and perform some analysis on the resultant, much smaller table, whose advantage consists in the fact that it is much better filled with data than the input table.

Information lost as a result of such truncation is of low importance and does not change the outcome in a significant way. However, it should be checked if this is true also for other proposed methods.

4 Summary

(4.0.1) The main goal was to construct the aggregated indicator based on 63 KPIs to enable comparison of the level of service rendered by TPW to all Alternative Operators. Proposed were two methods, namely the statistical one called Principal Component Analysis and the second technique that was based on the two easily calculated formulas.

(4.0.2) Suggested methods were implemented and some simulations using the real data set were performed. Based on the computations, rankings concerning the treatment of AOs were created. They could be used to identify discriminated alternative operators. Both methods indicate similar operators (especially among the “big” ones) which are at a disadvantage. Proposed function method is useful to properly detect discriminated operators. This was tested in the experiment described in the Appendix.

(4.0.3) The problem of missing data was also considered. Computations of the F indicator performed on different data subsets proved that it is sufficiently immune to the lack of data when it comes to the detection of the most discriminated operator. However, it should be ascertained if it is true for other proposed aggregated indicators.

(4.0.4) Still, quite many questions are left unanswered, which can be the subject of further research. For example, it should be verified whether the proposed aggregated indicators allows for the comparison of the level of service in different time periods. Another important issue to consider is robustness i.e. the stability of aggregated indicators when input data changes insignificantly. The input matrix has many undefined cells, which raises the question of how to deal with *null* values (e.g. consideration of only smaller subsets of the input data, elimination of the *null* values or application of methods for filling empty cells).

Appendix A

Simulation experiment

A.1 The Rules

(A.1.0) In this section, we present the experimental simulation. It was performed because there was no proper data to test the proposed indicators. The purpose of this simulation was to verify the actual discrimination. Another reason behind this experiment was to find out whether the proposed indicators could detect discriminated operator.

(A.1.1) The following input data for the simulation was chosen:
- 4 services and one KPI for each service;
- each KPI has a value from the range (0,1).

(A.1.2) Steps of the simulation:

1. AOs send requests for services. Operators:

- Type A ("big" operator) – there are 3 AOs: each has 3 services out of 4.
- Type B ("small" operator) – there are 5 AOs: each has 2 services out of 4.

2. The Incumbent selects a discriminated AO (Type A) and generates KPIs. The Incumbent chooses the strategy so that his intention would not be easily discovered. Only one AO of Type A must be discriminated.

3. Using the aggregated parameters F and Ord, the Regulator investigates AO which is discriminated.

4. It has to be checked whether aggregated parameters detect the discriminated AO correctly.

Table 4. Example of table that should be filled by Incumbent.

Operator	KPI 1	KPI 2	KPI 3	KPI 4
1 A (3 KPIs)				
2 A (3 KPIs)				
3 A (3 KPIs)				
4 B (2 KPIs)				
5 B (2 KPIs)				
6 B (2 KPIs)				
7 B (2 KPIs)				
8 B (2 KPIs)				

A.2 Example of the simulation

(A.2.1) Operator AO2 was deemed as discriminated. AO is discriminated if it has lower KPI than another AO in the market (with difference at least 0.1) for at least two KPIs. Table 5 is filled in the following way:

Table 5. Example of filled table.

Operator	KPI 1	KPI 2	KPI 3	KPI 3
1 A (3 KPIs)	0,95	0,8	0,59	
2 A (3 KPIs)	0,85		0,5	1
3 A (3 KPIs)		1	0,59	1
4 B (2 KPIs)	0,95			
5 B (2 KPIs)			0,6	1
6 B (2 KPIs)	0,8	1		
7 B (2 KPIs)		1	0,58	
8 B (2 KPIs)		1		1

Table 6.Results of indicators F and Ord for operators from Table 5.

Discrimination list (operator)	F	Discrimination list (operator)	Ord
AO2	0,556	AO2	0,037
AO6	0,500	AO1	0,027
AO1	0,500	AO6	0,024
AO7	0,375	AO5	-0,014
AO3	0,083	AO3	-0,019
AO8	0,000	AO8	-0,020
AO5	0,000	AO7	-0,024
AO4	0,000	AO4	-0,063

(A.2.2) Table 6 demonstrates the order of AOs discrimination by means of values of the aggregated parameters F and Ord. The higher position of AO in the list, the more discriminated it is.

According to both aggregated parameters, AO2 is the most discriminated AO in this example. This result is in line with the choice of the Incumbent.

A.3 Conclusions of the simulation

(A.3.1) The simulation was run eleven times and eleven tables with KPIs were created.

(A.3.2) In order to hide discrimination of a “big” AO, the Incumbent often discriminates smaller operators. As a result, in some cases a “small” AO is even more discriminated than the “big” one.

(A.3.3) Using the aggregated parameters F and Ord, the discriminated “big” AO was identified correctly by either F or Ord aggregated indicator in all cases. When using only parameter F, we obtained a wrong result once; whereas when using only parameter Ord - three errors occurred.

Bibliography

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