AIR HANDLING SYSTEM OPTIMISATION

Often each floor of a building has an air handling system consisting of a plant room, ducting and equipment such as fans and heaters. The cost of the system consists of the capital cost of the equipment and the operating cost to satisfy the thermal loads. An efficient method is required for evaluating the operating costs when the configuration of the system is specified and the thermal loads are known. The operating cost of a particular configuration are obtained by solving a nonlinear program. The method is efficient since it consists of solving a sequence of single period models.

1. Introduction

This problem was presented to the 1991 Mathematics-in-Industry Study Group at the University of South Australia by Kinhill Spence. The problem is concerned with the air conditioning of buildings. The decisions which need to be investigated are the configuration of the ducting for a floor of the building and the optimal configuration of secondary devices in each zone. Presently a computer package called BUNYIP is used to evaluate the operating cost for a particular configuration of air handling equipment. An alternative method is required which speeds up the evaluation.

A typical air handling system is sketched in Figure 1. A fan forces a mixture of fresh air and recirculated air through primary heating and cooling coils. Then the air passes along a duct to a zone on the floor of the building. Each zone may have secondary devices which reduce the volume of air, mix the air with heated air in the ceiling, or reheat the air. There is no provision in the zones for cooling of the air. The air from the zones returns to the fan via the ceiling cavity, where additional heating of the air takes place.

In the normal design process there are a large number of possible configurations of ducting which link the zones to the primary air handling plant. For each configuration there are many possible ways of configuring the secondary devices.

Kinhill Spence is therefore interested in methods which efficiently compare alternative air handling configurations. They are prepared to specify the thermal loads in each zone during each time period. They require an efficient method of calculating the operating cost of the configuration for specified thermal loads.

Heat transfer into a zone is defined as a positive heat transfer, while heat transfer out of a zone is defined as a negative heat transfer. The zone with the

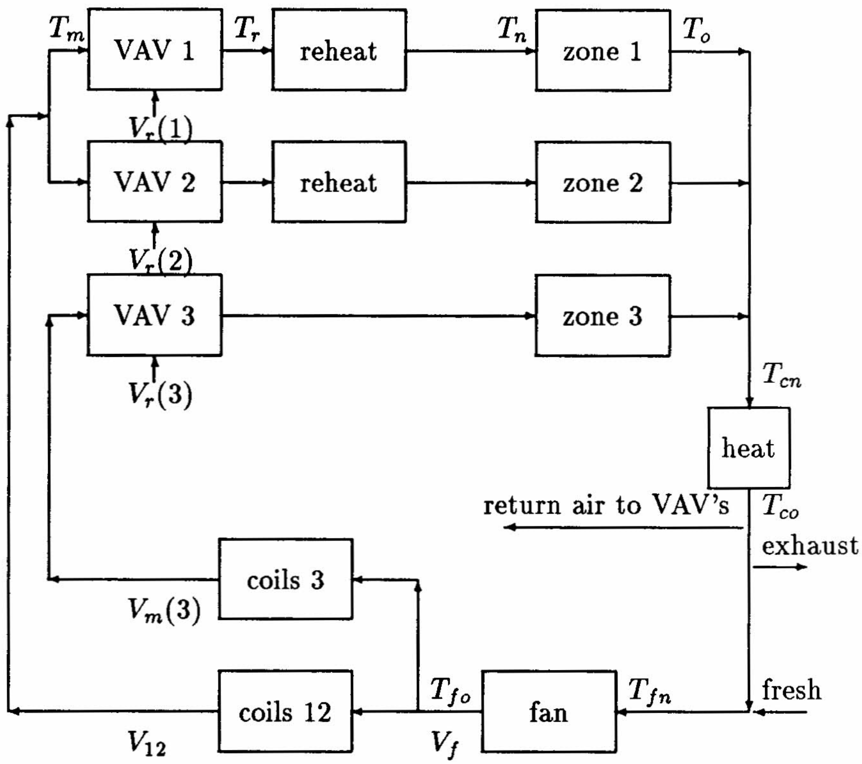

Figure 1: Configuration of an air handling system.

highest thermal load controls the air flow in the duct so as to keep the zone at its upper temperature limit for comfort. Other zones on the duct also receive air which is controlled by this zone.

If the supply air to a non-controlling zone receives air which produces a temperature below its lower temperature limit of comfort, then several actions can take place. The volumetric flow rate of air into the zone may be reduced by using a variable air volume (VAV) box. If the air reaches its minimum allowed volumetric flow rate without the temperature reaching the comfort range, then the air may be mixed with return air from the ceiling space. If the temperature in the zone is still below the lower limit, a reheat device is used to heat the air. Generally the priority order for introducing secondary devices is VAV, VAV with return air, and finally reheat.

A thorough examination of the design process involves time dependent heat flows and humidity effects. A full treatment of all these effects was impossible during the Study Group. The crucial issue for the mathematicians was to develop an efficient method of comparing alternative configurations of ducting and secondary devices. A good way to investigate the problem was to look at steady state conditions without humidity effects for a specified scenario of thermal loads and ambient temperatures.

Given an air handling configuration and the load profile over a whole year, the objective is to find the operating cost. This report shows that mathematical programming methods can efficiently compare various configurations of air handling equipment. The input to the model is a sequence of thermal loads and ambient temperatures. The output is the operating conditions and operating cost in each period.

Section 2 discusses the process of designing the air handling equipment for a floor of a building. The types of equipment are listed and their function is discussed. Section 3 develops a mathematical model of the air handling equipment. The notation is introduced and a formulation is given of the air handling problem. The method of solving the model is discussed. Section 4 uses a numerical example which illustrates the air handling model. Mathematical programming methods are used to determine controls and flows. Section 5 discusses the results discovered by the experiments shown in Section 4.

2. Background

Figure 2 illustrates the processes involved in the design of an air conditioning system for a building. The architect designs the shape of the building on the basis of functional requirements and the constraints of the allotment. Generally each floor of the building has an air handling plant consisting of fresh air intake, fan, and cooling and heating coils. Ducting and flexible tubes are used to distribute air. The ceiling cavity is used for return flow to the plant.

Each floor is divided into a number of separate zones on the basis of the thermal loads. The exterior of the building is referred to as the skin. The region within 4 metres of the skin forms the perimeter zones. Often these zones consist of individual offices with window outlooks. The remainder of the floor area forms an internal zone. Often this zone is used for open plan activities.

Each zone has an annual thermal load profile which is determined from factors external to the building such as expected weather conditions and internal factors such as occupancy and lighting. It may be possible to characterise the load profile using peak and marginal days in each of summer, mid-season and winter. Because the system is designed for peak loading it is often found that problems occur in mid-season. There may be a large variation in thermal loads between zones served by the same duct.

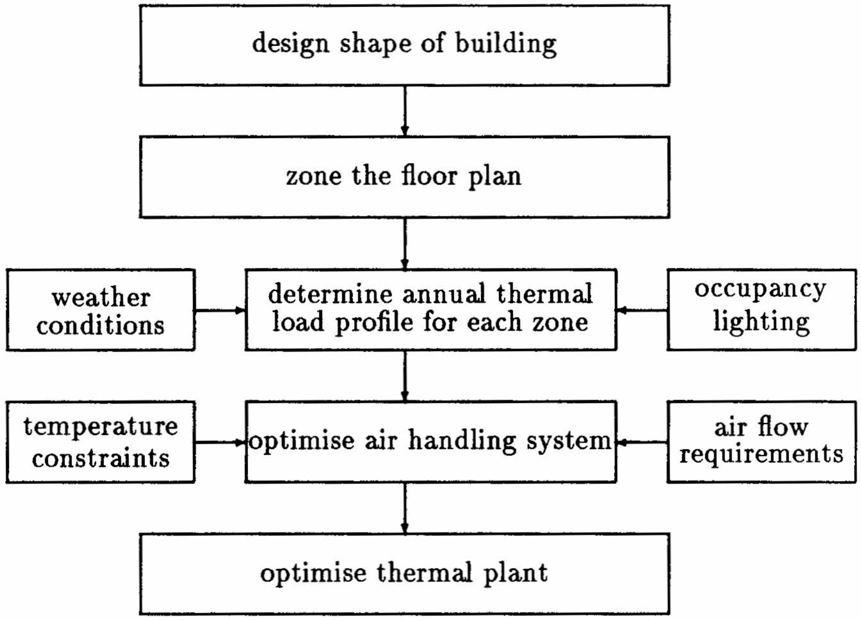

Figure 2: Processes involved in the design of an air-conditioning system.

Room comfort is measured by temperature and air flow. The typical temperature range is 20°C to 25°C and the minimum air flow rate of 5 litres per second per square metre. A minimum fresh air requirement is 5 litres per second per person. The percentage of fresh air may be as high as 100% if the ambient conditions are suitable. We assume a floor plan with ducting in the ceiling for cooling and heating.

Primary air handling equipment provides conditioned air to each zone. The air handling equipment supplies cooling and heating to satisfy the thermal load in each zone within a comfort tolerance for temperature and air flow. There are an enormous number of possible configurations of air handling equipment which will satisfy the load profile for each zone. A configuration with separate ducting to each zone has a high capital cost and low operating cost. A configuration with one line of ducting, variable volume boxes, and reheat coils has a low capital cost and high operating cost.

A VAV box uses adjustable dampers to control the flow of air through the box. Each diffuser maintains its proportion of the air flow as the volume changes. A fan assisted VAV box is able to mix return air with air from the duct to change the temperature of the air in the box. Heaters are used to reheat air from the duct that is too cold.

The air temperature in the duct is determined by the zone with the maximum demand for cooling. This is because there is no capacity for cooling in the secondary distributed devices. Cooling capacity resides in the floor air handling plant. Reheat facilities are available in the secondary devices. Once an air handling system is specified, the thermal load profile for the zones determines the requirements of the air handling plant. The sum of these load requirements for each floor determines the demand requirements on the building thermal plant. This demand profile is used to optimise the design of the thermal plant (Tostevin, 1982). This thermal plant is often located in the basement or roof and consists of a boiler and a chiller plant. Some thermal plants produce ice overnight under off-peak tariffs for use in the daytime peak load period.

Currently the design of the thermal plant can be optimised, but the same cannot be said for the air handling systems generating the loads. A method is required to evaluate the life-cycle cost of the specified configuration for given external and internal thermal loads. There is a trade-off between centralised or distributed equipment for cooling, heating and control.

The engineer accepts the building shape. The floor plan is partitioned into zones such that the thermal load can be calculated for each zone. Weather data and building equipment are used to determine the annual thermal load for each zone. A ducting configuration is designed to distribute air to each zone. It is relatively simple to evaluate capital cost of the air handling plant and ducting. It is more difficult to evaluate the operating cost of the configuration in response to the annual zone load profile.

Kinhill Spence uses BUNYIP (Moller and Wooldridge, 1985) to develop a cooling, heating and electrical supply profile for the air handling system. BUNYIP does allow some investigation of alternative air handling systems. However a more efficient method is required which facilitates comparison of alternative configurations.

3. A model for an air handling system

There are many textbooks which discuss air conditioning (McQuiston and Parker, 1982), the physics of heat transfer in buildings (Billington, 1967) and the energy balance equations in an air conditioning system (Eastop and McConkey, 1988). This report considers the volumetric flow rate and temperature of the air but ignores humidity aspects.

The assumptions used in the model are as follows

1. the system is in steady state in each period

2. there are no storage effects between periods

3. each zone is independent of the other zones

4. for each primary zone in the central plant, either the heater or the cooler is used, but not both in a period

5. the effects of humidity are ignored in the analysis

6. there is a requirement that a proportion, α, of the recirculating air be fresh air

Consider the air handling system for a floor of a building as a plant, ducting and secondary control devices. The plant consists of a fan and primary heating and cooling coils.

Suppose there are n zones in the building which are indexed by $j = 1, ..., n$. In zone j at time i the temperature is denoted by $T(i,j)$, the air volumetric flow rate by $V(i,j)$, and the thermal load by the heat transfer rate $Q(i,j)$. The SI units are Kelvin, litres/s, and Watts respectively.

The following notation is used to denote the constants in the air handling system.

$Q(i,j)$	thermal load in a zone (kW)
$T_a(i)$	ambient temperature in a period (^{o}C)
c_v	specific heat (J/litre ^{o}C)
G	heat input in the ceiling (kW)
V_l	minimum volume flux of fresh air (litre/s)
c_f	cost of operating the fan (\$/(litre/s))
c_h	cost of operating the heater (\$/kWH)
c_c	cost of operating the cooler (\$/kWH)
c_e	cost of operating secondary reheat (\$/kWH)

The following notation is used to denote the variables in the air handling system.

$T_m(i,j)$	temperature at inlet to secondary device (^{o}C)
$T_r(i,j)$	temperature of return air to secondary device (^{o}C)
$T_n(i,j)$	temperature at inlet to zone (^{o}C)
$T_o(i,j)$	temperature at outlet to zone (^{o}C)
$T_{cn}(i)$	temperature at inlet to ceiling (^{o}C)
$T_{co}(i)$	temperature at outlet to ceiling(^{o}C)
$T_{fn}(i)$	temperature at inlet to fan (^{o}C)
$T_{fo}(i)$	temperature at outlet to fan (^{o}C)
$V_m(i,j)$	volume flux at inlet to secondary device (litre/s)
$V_r(i,j)$	volume flux of return air to secondary device (litre/s)
$S(i,j)$	heat input to secondary device (kW)
$H(i,j)$	heat input to primary coils (kW)
$C(i,j)$	primary cooling (kW)
α	proportion of fresh air
z	total operating cost ($)

The input data is the thermal load in each zone, the ceiling heat load, the fresh air ambient temperature and the costs of energy.

The equation governing the energy flow balance is

$$Q = Mc(T_o - T_n) \tag{1}$$

where Q is the heat transfer (kW), M is the mass flow rate (kg/s), c is the specific heat of the air (kJ/kg °C), and $\Delta T = T_o - T_n$ is the temperature difference (^{o}C) between the outlet and inlet of the zone.

The energy flow balance can be expressed in terms of the volumetric flow rate of moist air as

$$Q = V s_h(T_o - T_n) \tag{2}$$

where V is the volumetric flow rate (litre/s) and s_h is the sensible heat of the moist air (kJ/litre °C). The sensible heat is the ratio of the specific heat of moist air to the specific volume of moist air at 21°C and 50% relative humidity. Its value is 0.001213 kJ/litre °C.

If a range of temperature is specified at the room outlet, then there are many feasible solutions. The optimal solution is determined by the following nonlinear programming problem:

$$\min z = c_f V_f + c_h(H(12) + H(3)) + c_c(C(12) + C(3)) + c_e \sum_j S(j) \tag{3}$$

subject to

$$0.0012 V_f \quad = \quad V_f s_h(T_{fo} - T_{fn}) \tag{4}$$

$$
\begin{aligned}
H(12) - C(12) &= V_{12}s_h(T_m(12) - T_{fo}) && (5)\\
H(3) - C(3) &= V_m(3)s_h(T_m(3) - T_{fo}) && (6)\\
S(j) &= (V_m(j) + V_r(j))s_h(T_n(j) - T_r(j)) && (7)\\
Q(j) &= (V_m(j) + V_r(j))s_h(T_o(j) - T_n(j)) && (8)\\
G &= V_c s_h(T_{co} - T_{cn}) && (9)\\
V_f &= \sum_j V_m(j) && (10)\\
V_c &= \sum_j (V_m(j) + V_r(j)) && (11)\\
V_{12} &= (V_m(1) + V_m(2)) && (12)\\
V_m(j) + V_r(j) &\leq 300 && (13)\\
V_m(j)T_{12} + V_r(j)T_{co} &= (V_m(j) + V_r(j))T_r(j) && (14)\\
V_cT_{cn} &= \sum_j (V_m(j) + V_r(j))T_o(j) && (15)\\
T_{fn} &= \alpha T_a + (1 - \alpha)T_{co} && (16)\\
\alpha V_f &\geq V_l && (17)
\end{aligned}
$$

The dependence on the index i is suppressed because the time periods are independent when calculating operating costs. Operating costs and heat inputs from VAV fans are ignored.

Equation (3) is the total operating cost of the fan, heating coils, cooling coils and reheat devices. The left hand side of equation (4) is the heat generated by the primary fan, and is a polynomial function of V_f. For simplicity we have used only the linear term. Equations (4) to (9) are energy flow balance equations for the fan, primary heating and cooling coils, secondary reheat devices, zones and ceiling. Equations (10) to (13) relate the volume flow rates, while equations (14) to (15) relate the mixing of the air at different temperatures. Equations (16) to (17) relate to the addition of fresh air to the system.

We used GAMS software (Brooke, Kendrick and Meeraus, 1988) to investigate this model on a PC. Tolerances and options were left at their default settings.

4. Results of a test problem

The input data includes the thermal load in the zone and the fresh air ambient temperature as a function of time. Table 1 shows the data used in the simple illustrative example with two time periods.

The time independent input data is the ceiling heat load, the minimum

Table 1: Input data for the simple model

Q(1)	3.50	-3.00
Q(2)	1.50	2.00
Q(3)	2.00	-2.00
T_a	30	10

volumetric flow rate and the energy costs. Table 2 shows the data used in the example.

Table 2: Constants for the simple model

s_h	1.213E-3
G	1.00
V_l	50
c_h	0.03
c_c	0.04
c_e	0.11
c_f	1.65E-4

The operating cost was determined for three configurations of the example problem. The first configuration had a variable speed fan and secondary reheat devices in zones 1 and 2. The results for this configuration are given in columns 2 and 3 of Table 3. Output temperatures in each zone are at their limits and the secondary reheat device is used in zone 1 during period 2. The operating cost for two periods (hours) is $0.94.

The second configuration has a fixed speed fan and secondary reheat facilities in zones 1 and 2. The results are given in columns 4 and 5 of Table 3. The air flow rate is at its maximum in each zone for each period and the outlet temperatures are at their limits. The secondary reheat device supplies heat to the cooler of zones 1 and 2. The operating cost for two periods is $1.13. Although the operating cost for the fix speed fan is more than that of the variable speed fan, it is the cheaper solution when the capital cost of the variable speed fan is included.

The third configuration has a variable speed fan, return air to a VAV box in each zone and reheat devices in zones 1 and 2. In addition the outlet temperature at the primary cooling coils is constrained to greater than 13°C. The results given in columns 6 and 7 of Table 3 show that significant return air is used for heating and the operating cost is $0.88.

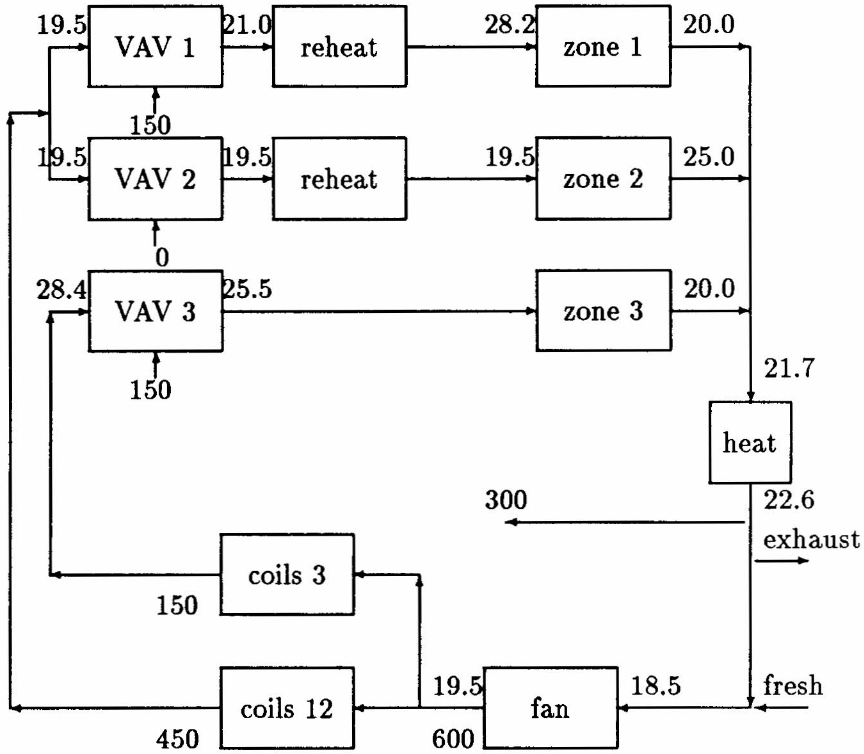

Figure 3: Results for the third configuration during the second period where values with a decimal point are temperatures (°C) and other values are volumetric flow rates (litres/s).

5. Discussion

The model given in Section 3 requires only the specification of the thermal loads and the ambient temperatures for each period. Very little time is required for the data preparation stage.

The results shown in Section 4 indicate that the warmest zone on a common duct is at its upper temperature limit in every period. For the variable speed example, zone 1 is warmer than zone 2 in period 1 and its outlet temperature is 25°C, while zone 2 is warmer than zone 1 in period 2 and its outlet temperature is 25°C.

Good initial values for the variables reduce the computation time of the program. There are a number of ways for developing a good starting point.

The initial values can be made conditional on the thermal loads and ambient temperature for the period and the zone. An alternative method is to start from the optimal solution of a problem with similar thermal loads and ambient temperatures.

Table 3: Optimal values for the variables for the simple model

Example	Variable Speed		Fixed Speed		Return Air	
Period	1	2	1	2	1	2
$T_m(1)$	11.76	19.50	15.38	19.50	13.00	19.50
$T_r(1)$					13.00	21.04
$T_n(1)$	11.76	36.56	15.38	28.24	13.00	28.24
$T_o(1)$	25.00	20.00	25.00	20.00	25.00	20.00
$T_m(2)$	11.76	19.50	15.38	19.50	13.00	19.50
$T_r(2)$					17.87	19.50
$T_n(2)$	11.76	19.50	15.87	19.50	17.87	19.50
$T_o(2)$	20.00	25.00	20.00	25.00	22.90	25.00
$T_m(3)$	14.01	30.99	19.50	25.49	14.01	28.41
$T_r(3)$					14.01	25.49
$T_o(3)$	25.00	20.00	25.00	20.00	25.00	20.00
T_{cn}	23.55	22.50	23.33	21.67	24.18	21.67
T_{co}	25.14	23.87	24.24	22.58	25.48	22.58
T_{fn}	25.71	18.51	24.82	18.51	25.90	18.50
T_{fo}	26.70	19.50	25.81	19.50	26.90	19.50
$V_m(1)$	217	150	300	300	240	150
$V_m(2)$	150	300	300	300	150	300
$V_m(3)$	150	150	300	300	150	150
$V_r(1)$					0	150
$V_r(2)$					96	0
$V_r(3)$					0	150
$S(1)$	0	3.09	0	3.18	0	2.62
$S(2)$	0	0	0.18	0	0	0
$H12, C12$	-6.6	0	-7.6	0	-6.59	0
$H3, C3$	-2.3	2.1	-2.3	2.18	-2.35	1.62
α	0.116	0.386	0.100	0.323	0.100	0.323
Cost	0.94		1.13		0.88	

For a given configuration the calculations can be performed sequentially in time with the optimal solution of one period becoming the starting point for the next period. After the calculations for the final period are finished, the capital cost of the equipment can be included on the basis of the maximum value attained by the equipment.

The preliminary results indicate the model can determine the operating cost of a variety of secondary devices. The model compares the cost of various configurations of secondary devices for a particular configuration of ducting.

The model can be reformulated to include capital costs. This requires the introduction of binary integer variables and results in a mixed integer nonlinear program. Such models can be solved by decomposition methods (Benders, 1962).

Further work is require to model other configurations such as a constant volume 'skin' conditioners with a VAV serving other zones. It may be possible to evaluate such a configuration by fixing the value of a flow volume variable at a constant value.

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