STRATIFICATION IN STEELMAKING LADLES

The problem of temperature stratification in a steelmaking ladle is considered. There are three distinct zones in the flow, the wall boundary layer, the bottom stagnation zone and the central plug flow. Typical length, velocity and time scales are determined for the flow and compared to numerical simulations. A model of the wall boundary layer using similarity solution techniques is detailed. Models for the temperature stratification in the bottom stagnation zone and the plug flow are included. Recommendations to BHP for reducing the temperature stratification in the ladle and improvements to their numerical simulation are made.

1. Introduction

A steelmaking ladle is essentially a bucket which is used to take molten steel from the primary steelmaking area to the casting area. BHP uses them at all of its main steelmaking centres – RBPD (Newcastle, NSW), SPPD (Port Kembla, NSW), LPD (Whyalla, SA) and NZS (New Zealand). Up to 10 ladles may be in use at any one centre at a given time.

Ladles are generally cylindrical, with an open top, and a small offcentre casting nozzle in the base. They consist of an external steel shell (50–100 mm thick), and several layers of internal refractory lining (up to 400 mm). A refractory-lined steel lid can be used to cover the top of the ladle. Some general ladle data and physical data are shown in the table on p. 88.

Whilst molten steel is contained in a ladle, it loses heat to the refractories in the wall and base. Very little heat is lost through the top surface, due to an insulating slag layer. Natural convection causes the steel to circulate, flowing down near the walls, and rising in the centre of the ladle. Under normal operating conditions, a vertical temperature gradient forms in the ladle. This thermal stratification is undesirable. It generally causes variations in casting temperature (degrading caster performance), but in extreme cases, results in the steel solidifying in the casting nozzle. Such "freeze-ups" are both expensive and dangerous.

The problem presented was to construct a model of thermal stratification in the ladle. The uses of such a model are twofold. Firstly, to predict the degree of stratification for a particular set of operating conditions. Secondly, to allow general operating guidelines to be modified to minimise stratification.

Ladle dimensions

Height, H, 2 to 4 m
Internal radius 1.2 to 1.8 m
Capacity 65 to 275 tonnes

Operational data

$T_{initial} = 1850$ K
Wall flux, q_w, 50 to 150 kW/m^2
Base flux, q_b, 30 to 100 kW/m^2
Cycle time ~ 6 hours
Life ~ 50 cycles

Steel properties

density, $\rho = 7000$ kg/m^3
heat capacity, $c_p = 627$ J/kg.K
thermal conductivity, k = 28.8 W/m.K
viscosity of steel, $\mu = 5.5 \times 10^{-3}$ kg/m s
coeff. of thermal expansion, $\beta = 14 \times 10^{-5}$ K^{-1}

2. Observations from previous modelling

BHP Research has previously developed a full numerical model of ladle stratification. This model used the PHOENICS code to simulate transient two and three dimensional turbulent fluid flow and heat transfer. The predictions of this model indicate the ladle can be divided into three regions. Firstly, a downward flowing boundary layer forms near the ladle wall. Secondly, a large upward flowing zone forms over most of the rest of the ladle. Thirdly, a relatively quiescent zone forms in the base of the ladle. This zonal system is shown in figure 1.

The flow in zone 2 is almost totally axial. Further, the axial speed is independent of radial position for much of the time. This is referred to as plug flow. The zone may extend for up to 90% of the ladle radius. Heat transfer in this zone is dominated by convection.

Zone 3 is dominated by conductive heat loss to the ladle base. This zone is quiescent and stable due to the positive thermal expansion coefficient of molten steel. Its axial extent increases slowly with time. In this zone, temperature is almost independent of radius. Heat transfer is essentially one dimensional.

The one-dimensional nature enjoyed by zone 2 and by zone 3 suggests a simple model could be found. This model would be expected to be transient and one dimensional. It should show 2 main regions. Firstly, a base region corresponding to zone 3,

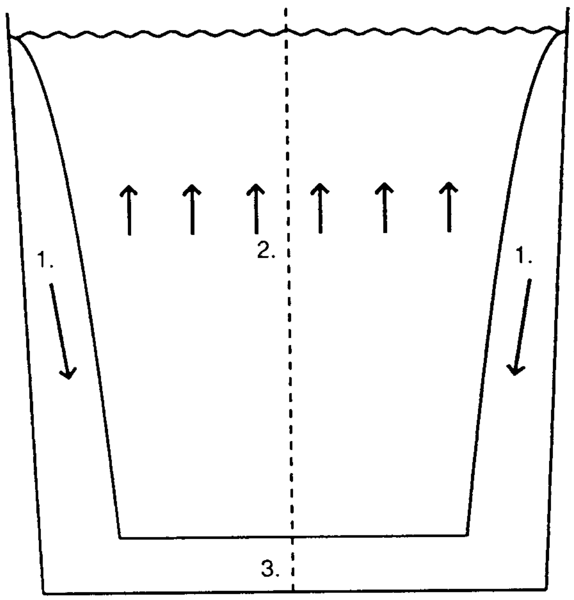

Figure 1: Zones of flow predicted to form in a ladle under natural convection: 1 is the wall boundary layer zone, 2 is the central or core zone, and 3 is the quiescent zone.

where conduction dominates. Secondly, a larger upper region where convection dominates, corresponding to zone 2. A mixing region might also appear between the lower and upper regions.

Of interest is the observation by Chakraborty and Sahai (1992) that when no slag is present, the recirculation can be strong enough to prevent a quiescent zone from forming. Could a minimum/critical wall heat loss rate be found to cause the same effect?

3. Overall heat balance

To comprehend the numerical results presented by BHP we need to first formulate an overall heat balance. As heat is lost through the wall and the base, the equation is

$$mc_p \frac{dT_{av}}{dt} = -(A_w q_w + A_b q_b)$$

where A_w and A_b are the wall and base areas respectively, and q_w and q_b are the wall and base heat fluxes respectively.

This equation can be rearranged to make $\frac{dT_{av}}{dt}$ the subject. It is this quantity, the rate of change of the average temperature of the molten steel in the ladle, which BHP have used as their measure of the total heat loss. Furthermore, the graphical results presented by BHP to the Study Group used this quantity, and were of the form

$$\frac{d(Stratification)}{dt} = Const\frac{dT_{av}}{dt}$$

where *Const* is a non-dimensional number and stratification is defined to be $T_{top}-T_{bottom}$. For one particular ladle, *Const* = 2.

4. Length, velocity and time scales

The numerical results and physical arguments suggest that the cooling at the wall produces a wall boundary layer and that this drives the mass flux in the ladle. In particular, the flow in the core should be the direct result of the flow in the wall boundary layer.

Presuming this to be the case, it is vital to determine an estimate of the time required for the wall boundary layer to form and to compare this with the time required for a complete mass exchange through the boundary layer. Additionally, we would hope to be able to estimate the time needed for the formation of any stagnant layer at the base.

As a first approximation the vertical momentum in the wall boundary layer balancing the buoyancy gives a typical boundary layer velocity, u, given by

$$\frac{u}{H} \sim \beta g \Delta T \tag{1}$$

where ΔT is the change in temperature across the boundary layer. Similarly the heat into the boundary layer due to advection balances the heat out of the boundary layer due to diffusion.

$$\frac{u}{H} \sim \frac{\kappa}{\delta^2} \tag{2}$$

where δ is a characteristic width of the boundary layer. The case under consideration is of a constant heat flux into the refractory lining; hence

$$q_w \sim k\frac{\Delta T}{\delta} \tag{3}$$

Eliminating u and ΔT from (1-3) gives

$$H \sim \frac{\beta g q_w \delta^5}{k\kappa^2}$$

From this an estimate of a typical boundary layer thickness, δ, can be determined in terms of a modified Rayleigh number (compare with Rohsenow *et al.*, 1985 pp. 6–17)

$$Ra^* = \frac{\beta g q_w H^4}{k\kappa^2}$$

and the height of the ladle as

$$\delta \sim HRa^{*-\frac{1}{5}}$$

Taking a typical value of the wall flux, q_w, of 100 kW/m^2; a height, H of 4m; and the thermal properties defined in section 1 then

$$\delta \sim 0.0081 \text{ m}$$

The characteristic velocity given by (2) is

$$u \sim \frac{\kappa}{H} Ra^{*\frac{2}{5}}$$

With the values used here we find

$$u \sim 0.397 \text{ m/s}$$

As this is only a first approximation, these are representative values for the boundary layer thickness and velocity where the bulk of the boundary layer flow occurs. The actual boundary layer thickness is expected to be somewhat thicker than this value, with a generally lower velocity, since there is a smooth transition to the interior flow. A typical temperature difference across the boundary layer can also be written in terms of the modified Rayleigh number. From (3)

$$\Delta T \sim \frac{Hq_w}{k} Ra^{*-\frac{1}{5}}$$

which, for the typical values used previously, gives

$$\Delta T \sim 28.2°\text{C}$$

The volume of the boundary layer and the characteristic velocity yield an estimate of the mass flux:

$$\frac{dm}{dt} = 2\pi R \delta u \rho \sim 213 \text{ kg/s.}$$

Therefore the characteristic core velocity is determined:

$$v = \frac{2\delta u}{R} \sim 4.3 \text{ mm/s}$$

These results are encouraging as they are of a comparable magnitude as the numerical results obtained using the PHOENICS code.

Having estimated the length and velocity scales, we are now in a position to estimate the all important time scales. Firstly, the time for the thermal boundary layer to be established:

$$\tau_{thermal} \sim \frac{\delta^2}{\nu} \sim 80\text{s} = 1.3 \text{ min}$$

and secondly, the time scale for the core flow:

$$\tau_{core} \sim \frac{H}{v} \sim 15.5 \text{ min}$$

We can also estimate the time required for a balance between advection and conduction in the stagnant layer in the base:

$$\tau_{base} \sim= \frac{\kappa}{v^2} \sim 0.35 \text{ s}$$

and the height at which this balance occurs:

$$h \sim \frac{\kappa}{v} \sim 15 \text{ mm}$$

Finally, the time scale for conduction in the refractory is of interest, for a refactory of 100mm

$$\tau_{refractory} \sim \frac{L^2}{\kappa_{refractory}} = 220 \text{ min}$$

As this last number is larger than the average waiting time for ladles (around 20 minutes), the insulation should remain relatively cool and we need not be concerned with the specific details of the insulation layer.

To conclude this section, it is worth emphasising that we have determined that the wall boundary layer is very quickly established, and that the ensuing transient behaviour in the ladle is quasi-steady.

5. Temperature stratification at the base

It was of interest to isolate the effect of heat loss from the base. Assuming a stagnant layer forms at the base, then near the ladle centreline the stratification is approximately a one-dimensional heat conduction problem with a prescribed flux. The solution can be obtained by elementary means, or found in textbooks; *e.g.* Rohsenow *et al.* (1985, pp. 4–108):

$$\Delta T = \frac{2\sqrt{\kappa t}}{k} q_b \text{ ierfc}\left(\frac{x}{2\sqrt{\kappa t}}\right)$$

where

$$\text{ierfc}(\eta) = \frac{1}{\sqrt{\pi}} e^{-\eta^2} - \eta \text{ erfc}(\eta)$$

The stratification as a function of time occurring solely due to this effect is given by

$$Stratification = \Delta T(H,t) - \Delta T(0,t)$$

As an example this can be evaluated for very small times for a 4 metre ladle to be

$$Stratification = \frac{2q_b\sqrt{\kappa t}}{k\sqrt{\pi}}$$

For a base heat flux of 100 kW/m^2, the stratification initially behaves like $10\sqrt{t}$, which represents a very quick temperature stratification.

Regardless of the precise details, this demonstrates that the base heat flux alone causes considerable stratification. Initial computations done by Peter Austin (of BHP Research) with the numerical code verified that eliminating the base heat flux would alleviate the stratification problem. This led to the suggestion that BHP consider putting more effort into the insulation layer at the base.

6. The wall boundary layer in detail

The natural convective flow induced by cooling (or heating) a wall is a much studied problem. In the absence of imposed flows, a boundary layer is set up at the wall. The (Prandtl) boundary layer equations are (Yih, 1969; Brand and Lahey, 1967)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \beta g(T - T_\infty) \tag{5}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} \tag{6}$$

These equations are defined with respect to a cartesian coordinate system where x measures the distance from the top of the wall (u is the component of velocity in the x direction) and y is a coordinate transverse to the surface (v is the velocity component in the y direction). The large radius of the ladles, compared with the (expected) size of the boundary layer makes it reasonable to approximate the cylinder wall as a flat plate. T_∞ is the ambient temperature far away from the heated surface (*i.e.* the interior of the ladle).

A general similarity solution to the system (4–6) can be found by firstly introducing a streamfunction ψ, automatically satisfying the continuity equation (4)

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$

and, secondly, making the transformations

$$\eta = ayx^{(\alpha-1)}$$

$$\psi = a\nu x^{\alpha} f(\eta)$$

$$\beta g(T - T_{\infty}) = a^4 \nu^2 x^{(4\alpha-3)} h(\eta) \tag{7}$$

In these transformations, a and α are undetermined parameters.

The ordinary differential equations which result are

$$f''' + \alpha f f'' - (2\alpha - 1) f'^2 + h = 0 \tag{8}$$

$$h'' + \alpha\sigma f h' + (3 - 4\alpha)\sigma f' h = 0 \tag{9}$$

In these equations, a dash denotes differentiation with respect to η and σ is the Prandtl number, ν/κ.

In order to solve these equations, boundary conditions representing particular physical situations must be formulated. Then a and α can be chosen to satisfy these boundary conditions.

For example, one would usually require "no slip" at the wall:

$$y = 0 : v = 0, u = 0$$

Also, it would be expected that there is no temperature rise far away from the wall, and consequently no flow:

$$y = \infty : u = 0, T = T_{\infty}$$

In terms of f and h, these yield

$$f(0) = 0, f'(0) = 0, f'(\infty) = 0, h(\infty) = 0 \tag{10}$$

A well-posed problem still requires one further condition on $h(\eta)$, *or* on $h'(\eta)$. Brand and Lahey (1967) assume an insulated wall and are then able to determine α and solve the equations. The insulation condition is clearly inappropriate for the ladle. Instead, we wish to prescribe the heat flux at the wall, as BHP have done in their numerical simulations:

$$k\frac{\partial T}{\partial y}(y = 0) = -q_w$$

where q_w is the prescribed wall heat flux. In terms of the similarity form (7) for temperature, this means a condition on $h'(\eta)$:

$$ka^5 \nu^2 x^{(5\alpha-4)} h'(0) = -\beta g q_w$$

Requiring a constant heat flux from the wall fixes the value for $h'(0)$, and forces us to choose $\alpha = \frac{4}{5}$. We can further reduce the problem to canonical form by assigning a value to a:

$$a^5 = \frac{\beta g q_w}{k\nu^2}$$

The condition for $h'(0)$ is then

$$h'(0) = -1 \tag{11}$$

The boundary layer is then found by solving (8, 9) with $\alpha = \frac{4}{5}$; namely,

$$f''' + \frac{4}{5}ff'' - \frac{3}{5}f'^2 + h = 0 \tag{12}$$

and

$$h'' + \frac{4}{5}\sigma f h' - \frac{1}{5}\sigma f' h = 0 \tag{13}$$

subject to the boundary conditions given in (10, 11). The solution is best done numerically; and it only has to be done once, as all the information required for different operating conditions is included in a. Equations (12, 13) are written as a system of five first order differential equations and these are solved using a Runge-Kutta-Merson method with a Newton iteration in a shooting and matching technique (as given by the NAG routine D02HAF). The results are shown in figures 2(a) and 2(b), for $f'(\eta)$ and $h(\eta)$ respectively. Velocity and temperature contours are then found, and the stratification from the wall heat flux can be displayed graphically, for example in figures 3(a) and 3(b) for the extremes of the given wall heat flux, $q_w = 50\text{kW/m}^2$ and $q_w = 150\text{kW/m}^2$ respectively. The higher degree of stratification with the higher heat flux is evident. The central zone exerts a small influence on the wall boundary layer which can be accounted for with a modified boundary condition. The results are only slightly altered.

The results presented here for the wall boundary layer can be related back to the approximate results derived in section 4. The momentum boundary layer thickness is defined to be the region where the non-dimensional velocity, $f'(\eta)$, is greater than some percentage (commonly 1%) of the maximum velocity and similarly the thermal boundary layer thickness is where the non-dimensional temperature, $h(\eta)$, is greater than some percentage of the maximum temperature. Using a 1% level here gives the momentum boundary layer thickness as $\eta = 10.8$ and the thermal boundary layer thickness as $\eta = 11.6$. These are expected to be of a similar magnitude as the Prandtl number is small and the physical situation is one of free convection. Relating these to the actual variables gives a maximum boundary layer thickness of 0.034m for $q_w = 100\text{kW/m}^2$ at $x = 4$m. This is a factor of up to 4 times the boundary layer thickness predicted in section 4. This is not unexpected as the first order boundary layer estimates in section 4 are for the bulk flow (where the velocity and temperature gradients are the largest) and are a characteristic thickness over the entire boundary layer, not a maximum value. The maximum velocity from the full boundary layer solution (which is in the bulk flow region) is 0.390m/s which is in agreement with the characteristic velocity for the bulk

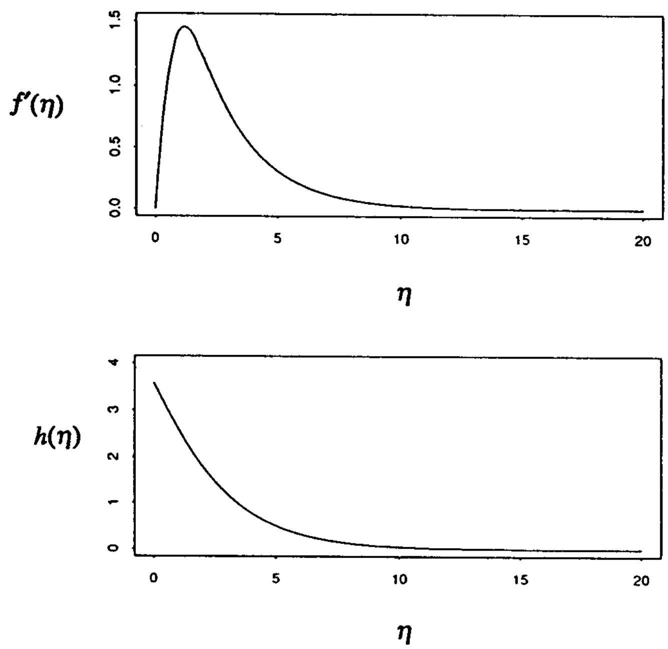

Figure 2: (a) Non-dimensional velocity $f'(\eta)$. (b) Non-dimensional temperature $h(\eta)$.

flow (0.397m/s). The maximum temperature difference across the full boundary layer is $36.2°C$ which is quite close to the typical temperature difference across the bulk flow region found in section 4 of $28.2°C$. Overall the approximations derived in section 4 give surprisingly good agreement with the full boundary layer solutions presented here and hence the predicted time and length scales are expected to be a reasonably good estimate.

7. A simplified flow model

The essential features of the stratification process seemed to be (i) the flow down the wall caused by the wall heat flux, (ii) the base heat flux, and (iii) the flow up the centre. For this reason it was deemed appropriate to formulate a one-dimensional model for the bulk motion of the ladle. This model has two temperatures: T_1 in the wall layer with downward flow at a velocity u; and T_2 in the centre with upward flow at a velocity v. A hyperbolic system then describes the advection of heat and the wall heat loss:

$$\frac{\partial T_1}{\partial t} + u\frac{\partial T_1}{\partial x} = -\frac{q_w}{\rho c_p \delta}$$

$$\frac{\partial T_2}{\partial t} - v\frac{\partial T_2}{\partial x} = 0$$

Coupling occurs through the boundary conditions at the top ($x = 0$) and the base ($x = H$), this must include any base heat flux, which is derived from the balance between

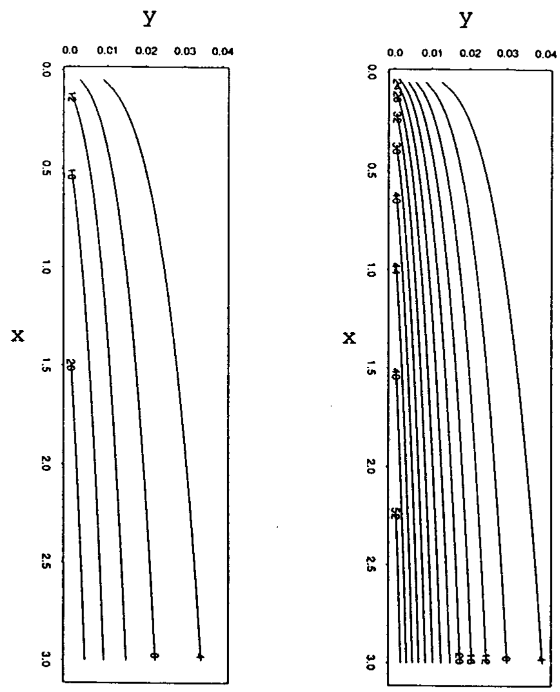

Figure 3: Temperature contours below the interior ambient temperature T_∞ (a) $q_w = 50kW/m^2$ (b) $q_w = 150kW/m^2$.

advection and conduction in the stagnant layer and the height at which this balance occurs:

$$T_1(0,t) = T_2(0,t)$$

$$T_1(H,t) = T_2(H,t) + \frac{\tau_{\text{base}} q_b}{\rho c_p h}$$

where τ_{base} and h are relevant time and length scales for the quiescent zone, as calculated in section 4.

Analysis of this system using characteristics, and using Laplace Transforms highlighted a problem with initial conditions namely, it was not possible to satisfy a general set of initial conditions for T_1 and T_2. It is thought that a modification is possible to alleviate this problem, but there was insufficient time at the Study Group to resolve these issues.

Ignoring the initial condition problem, the stratification predicted was independent of time, also suggesting that the model is too simple.

8. Improvements to the numerical method

The PHOENICS code used by BHP in their numerical modelling takes some six hours to run for one set of conditions. This was one of the primary reasons for seeking greater intuitive understanding and a simple model of stratification.

It was suggested that there are ways to improve the numerical performance. To explain these improvements, we consider here a semi-discretised form of the model. That is, we consider time to be continuous and we discretise the spatial variables. In this case the system of equations to be considered is an implicit system of ordinary differential equations:

$$\frac{dT}{dt} = f(t, T).$$

As the system is likely to be extremely stiff, "L-stable" numerical schemes are recommended. The backward Euler scheme currently in use is L-stable, but only of first order accuracy and hence is very computationally intensive. Instead we suggest the use of second order accuracy schemes, such as the predictor-corrector scheme (Fitzsimons *et al.*, 1992), the Gear scheme, the Bell scheme (Bank *et al.*, 1985), and the composite integration scheme (Carroll, 1989). Using a second order scheme will permit the use of much larger step sizes and a correspondingly reduced CPU time. It would be wise to strategically choose the size of the next time step after each iteration. All of the schemes suggested have been successfully used to solve time-dependent equations for semiconductor devices (Bank *et al.*, 1985; Carroll, 1989; Liu, 1991).

9. Conclusion

We have determined that the flow pattern in the ladle is very quickly established, and that a quasi-steady cooling process ensues. The respective roles of the heat loss from the base and the wall have been examined. It seems the base heat loss is the cause of considerable vertical stratification; BHP are already taking measures in this regard. The wall heat flux drives the circulation and hence primarily causes horizontal stratification. It is unlikely to be desirable to reduce the wall heat flux as much as the base heat flux, because of the mixing role of the wall heat flux.

Further work should examine the quiescent zone in more detail. It seems possible to set up approximate balance equations for this zone; coupled to the wall boundary layer and the central convective zone. This would allow good estimates of the time for formation and the size of the quiescent zone.

The bottom corner, where the wall boundary layer meets the base, deserves separate investigation. It is likely that boundary layer matching would be required to unravel the intricacies of the fluid flow and heat transport in this corner; as numerical codes require extremely fine meshes to fully resolve the behaviour in corners.

So, in summary, the numerical results obtained by BHP have been verified on heat and mass balance grounds, a good model for the wall boundary layer has been derived and provisional models for the plug flow and bottom stagnation zone have been detailed. The modelling work performed during the course of the meeting has been useful in determining many of the bulk properties of the flow in the steelmaking ladles. This has lead to a more detailed understanding of the phenomenon of temperature stratification which is of particular concern to BHP. From this information some suggestions have been made to BHP which may help in alleviating the stratification problem and aid in future modelling of the steelmaking process.

Acknowledgements

The moderator (Rod Weber) would like to thank Peter Austin of BHP Research for keeping us on track. A large cast of academics participated in this problem. It is a pleasure to thank Steve Barry, Fawang Liu, Neville Fowkes, Glenn Fulford, David Jenkins, Kerry Landman, Sean McElwain, Geoff Mercer, Rhys Newman, John Roumeliotis, Joe Steiner, Nick Stokes and Simon Watt for their assistance. In particular, Geoff Mercer's help with the computations and the report are gratefully acknowledged.

References

R.E. Bank, W.M. Coughran Jr., W. Fichtner, E.H. Grosse, D.J. Rose and R.K. Smish, "Transient simulation of silicon devices and circuits", *IEEE Trans. Elec. Dev.* Vol. ED-32, **10** (1985), 1992–2007.

R.S. Brand and F.J. Lahey, "The heated laminar vertical jet", *J. Fluid Mech.* **29** (1967), 305–315.

J. Carroll, "A composite integration scheme for the numerical solution of systems of ordinary differential equations", *J. Comp. Appl. Math.* **25** (1989), 1–13.

S. Chakraborty and Y. Sahai, "Effect of slag cover on heat loss and liquid steel flow in ladles before and during teeming to a continuous casting tundish", *Metallurgical Trans.* **23B** (1992), 135–151.

C.J. Fitzsimons, F. Liu and J.J.H. Miller, "A second order L-stable time discretisation of the semiconductor device equations", *J. Comp. Appl. Math.* **42** (1992), 175–186.

F. Liu, "Numerical analysis of semiconductor device equations", *PhD thesis*, (Trinity College, Dublin, Ireland, 1991).

W.M. Rohsenow, J.P. Hartnett and E.N. Ganic, *Handbook of heat transfer fundamentals*, (McGraw-Hill, 1985).

C.S. Yih, *Fluid mechanics: a concise introduction to the theory*, (McGraw-Hill, 1969).