

## BLENDING METHODOLOGIES IN TALC OPERATIONS

The problem posed by Western Mining Corporation involves finding a way of improving or optimising the utilisation of batches of lower grade talc when making up orders for products of different grades.

During the MISG a number of Linear Programming models were developed. These models addressed the problems of blending batches of talc for a single order and of blending to meet a series of orders for different products over a specified planning horizon. Preliminary versions of the models were tested using data supplied by Western Mining Corporation.

### 1. Introduction

The Three Springs talc mine is the largest world class talc deposit in the southern hemisphere. Western Mining Corporation has operated the mine since 1960 and exports talc through the port at Geraldton WA to customers in Japan, Europe and the Americas. The talc is used in the production of materials such as paper, paint and ceramics.

The talc is mined by conventional open cut methods. Rock is fractured in-situ using explosives, then loaded onto trucks using face shovels and backhoes. Ore is crushed, washed and sized prior to having dolomite contaminants removed by hand as it passes along a conveyor. Each daily production batch, of 300–500 tonnes, is sampled to obtain reflectance and chemical data. Reflectance indicates the brightness of the talc and the chemistry indicates the levels of the contaminants  $\text{CaO}$ ,  $\text{Fe}_2\text{O}_3$  and  $\text{Al}_2\text{O}_3$ .

The batches are stored as individual stakes, which are small (300 tonne) stockpiles of known grade, in one of four stockpile areas. Material from stakes that is suitable to meet the product specifications for a particular order is manually blended, or loaded directly into trucks and then put on a train which takes it to the port for stockpiling prior to shipping.

### 2. Problem description

The problem posed by Western Mining Corporation involves finding a way of improving or optimising the utilisation of stakes containing lower grade talc when making products of different grades.

The process for building stockpiles of different products at the port is as follows.

### 2.1 Mining and processing batches of talc

1. The talc is mined by conventional open cut methods. After mining, batches of talc, of around 300 tonnes, are processed by crushing, washing and removing contaminants by hand from the conveyor carrying the talc.
2. The talc is sampled at 90 second intervals at the end of the processing stage and two basic parameters, brightness, or light reflectance, and contaminant level are measured.

The brightness is measured using a scale of 0–100. Average talc has a brightness of around 85 and high quality talc is in the range 90+.

Contaminants from dolomites in the talc ore body are  $\text{CaO}$ ,  $\text{Fe}_2\text{O}_3$  and  $\text{Al}_2\text{O}_3$ . Typical contaminant levels are in the range 0.3%–1.0%.

Based on samples taken during processing, each batch of processed talc is assigned a grade, which is a measure of its brightness and contaminant level.

3. Each processed batch of approximately 300 tonnes is stored as an individual stake, of known grade, in one of four stockpile areas. The term stake is used because each batch is identified by a number written on a wooden stake which is driven into the pile.

### 2.2 Product stockpiles at the port

A product is defined in terms of a minimum grade of talc. The grade specifications for different products are shown in Table 1.

The products are stored as separate large stockpiles at the port. An order for a given product will be loaded for shipping from the relevant stockpile. The product stockpiles are replenished using trains to carry talc from selected stakes to the port.

### 2.3 The blending problem

The problem facing WMC Talc Operations is that currently the higher grade talc stakes at the mine are most often used to make up the different products. This practice results in a large and growing number of low grade stakes which are of little use or value to the company.

As an example, suppose there is a need to build up a port stockpile for a product order which has a grade specification of Brightness 90, and  $\text{CaO}$  contamination  $< 0.5\%$ .

This order could be filled by loading a train from stakes with brightness 92+ and contamination levels < 0.4%. This 'simple' method of making up a product results in an inefficient utilisation of the talc in the stakes since it uses scarce resources (higher grade talc) to meet the product specification.

One way of overcoming this problem is to blend talc from different stakes in order to achieve the required product specification. A good blending procedure would mix as much lower grade talc as possible with just enough high grade material to meet the product specifications.

When blending, the grade of the blend is the weighted average of the grades of the stakes in the blend. Thus, if 60 tonnes from a stake with brightness 90 and CaO level 0.5% is mixed with 40 tonnes from a stake with brightness 92 and CaO level 0.6%, the resultant blend will have a brightness of 90.8 and a CaO level of 0.54%.

Also, when blending to achieve a desired brightness level, it is not acceptable to mix stakes with widely different brightness levels. This is because if one stake with a very high brightness level is mixed with another stake which has a very low brightness level, then it will be possible to detect the low brightness material within the blend. It is therefore necessary to specify a limited range of acceptable stake brightness levels for each required product brightness.

If the talc in the stakes is not of a high enough grade to be used for a given product, then some of the talc can be reprocessed. Reprocessing the talc involves sending it back through the washer and picking out more of the lumps of contaminants. The procedure is called repicking and increases the cost of the end product.

Repicking can be used to lower the contaminant level of the ore, but it does not change the brightness of the material in the stake. In some cases, if there are no stakes with the required brightness for a product order, the company may have to modify its planned mining operations and try to mine to specification for that order.

Based on the processes involved in achieving the product specifications for a series of orders, the questions that the company brought to the MISG were:

- Given orders for different products, and a set of stakes of known grades at the mine, which stakes should be mixed in order to make the products. That is, is there an algorithm that can be used to determine which batches to load onto the train to achieve the required product specifications?
- Is it possible to develop a blending algorithm which would maximise the use of the low grade stakes? Such an algorithm would allow Western Mining

Corporation to minimise the build up of large numbers of low grade stakes, while efficiently utilising the brightness and contaminant characteristics of individual stakes to meet product specifications.

The group working on this problem at the MISG attempted to answer these questions via the development of the optimisation models described in the next section.

### 3. Optimisation models

Two optimisation models were developed during the MISG. The first model looks at the problem of selecting which stakes to use when filling a customer order. The second model looks at choosing stakes to fill monthly orders for the full range of products.

#### 3.1 Blending for a single order

This linear programming, or LP, model deals with the issue of choosing which unblended talc to use to fill a customer order. As far as possible, low quality talc should be used in preference to high quality talc while satisfying the customer's quality requirements.

An order is represented as a demand for  $D_p$  tonnes of a product  $p$  with quality specifications defined by four grade criteria. The grade criteria refer to the Brightness level (GE) of the product, its Calcium (CaO) level, its Iron ( $\text{Fe}_2\text{O}_3$ ) level, and its Alumina ( $\text{Al}_2\text{O}_3$ ) level. The product on order will have limits set on the values of each grade criterion. The unblended talc is arranged in stakes ( $s$ ) which are grouped in four stockpile areas ( $A$ ).

The sets, variables and parameters used in the model are as follows.

#### *Sets*

$S$  The set of all stakes  $s$ .

$\mathcal{H}$  A set of four stockpile areas  $A \subseteq S$ . The members of  $\mathcal{H}$  partition  $S$ .

The formulation of the basic single order model discussed below does not make use of the stockpile area sets  $A$ . The reason for partitioning  $S$  into areas  $A$  is that for certain variations of the basic model, which are discussed in Section 3.2, we may wish to add cost penalties to, or prevent the use of, stakes from specified areas.

*Variables*

$x_s$  Tonnes of stake  $s$  used directly for blending product  $p$ .

$y_s$  Tonnes of stake  $s$  repicked to achieve grade specifications for product  $p$ .

*Costs*

$C_s$  Per tonne production cost of talc in stake  $s$ .

$C_{RP}$  Additional per tonne cost for repicking talc.

*Parameters*

$D_p$  Demand (tonnes) for product  $p$ .

$B_s$  Brightness of stake  $s$ .

$O_s$  Calcium Oxide level of stake  $s$ .

$O_s^{RP}$  Calcium Oxide level of stake  $s$  after repicking.

$B_p^U$  Upper brightness limit for product  $p$ .

$B_p^L$  Lower brightness limit for product  $p$ .

$O_p^U$  Upper Calcium Oxide limit for product  $p$ .

$L_s$  Initial tonnage of stake  $s$ .

$R_p$  A brightness range specifying which stakes can be used to blend product  $p$ .  $R_p$  is calculated as a number of points above the upper limit and below the lower limit of the brightness of product  $p$ . We have  $B_s \in R_p$  for stakes within this brightness range.

Note that the model uses only the Brightness and Calcium limits to determine the product specification limits. These are the two most important criteria.

Values for the upper and lower brightness limits,  $B_p^U$  and  $B_p^L$ , and the upper Calcium Oxide limit,  $O_p^U$ , are given in Table 1. Values for the demand in the periods  $t = \text{Jan} \dots \text{Jun}$  are given for each product in Table 2. Table 3 shows the stockpile area and values for the brightness  $B_{sa}$ , Calcium Oxide content  $O_{sa}$  and tonnage  $L_{sa}$ , for the first 11 of 148 stakes  $s$ .

The value of  $O_{sa}^{RP}$  was set to  $0.9 O_{sa}$  during the tests of the model. This means that repicking material from a stake will reduce its Calcium Oxide contaminant level by 10%.

For the single order model discussed in this section, the demand would be specified by one of the values in Table 2. For example, we could use the model to determine how to blend the order for 8808 tonnes of product 2A in January, with the relevant values for the Brightness and Calcium limits taken from Table 1.

Product	EC	1A	1	2A	2	WF
Brightness						
Upper Limit	93	93	90	88	86	84
Lower Limit	87	90	88	86	84	79
CaO						
Upper Limit	0.3	0.6	0.6	0.6	0.7	0.8

Table 1: Product Data.

	JAN	FEB	MAR	APR	MAY	JUN
EC			5000			4000
1A			3000			4000
1	340	300	500	300	3300	3300
2A	8808		2000		2000	8000
2	5630	5130	10130	5130	8130	8630
WF	2000	3000	10000			3000

Table 2: Demand (Tonnes).

Stake	Stockpile Area	$B_{sa}$	CaO	Tonnes
000027	SP4	89.8	0.37	93
000033	SP4	88.1	0.60	110
000058	SP4	87.7	0.63	102
000083	SP4	87.3	0.95	85
000349	SP2	85.5	2.38	306
000616	SP3	83.9	1.32	103
000735	SP3	81.9	1.13	272
000761	SP1	85.1	2.19	82
000767	SP3	84.9	2.37	221
000834	SP4	84.9	1.18	204
000864	SP4	77.4	0.25	182
⋮	⋮	⋮	⋮	⋮

Table 3: Stake Data.

### 3.1.1 The basic single order LP model

The LP model is formulated as follows.

$$\text{Minimise: } \sum_{s: B_s \in R_p} (C_s \mathbf{x}_s + (C_s + C_{RP}) \mathbf{y}_s) \quad (1)$$

Subject to:

$$\sum_{s: B_s \in R_p} (\mathbf{x}_s + \mathbf{y}_s) = D_p \quad (2)$$

$$\sum_{s: B_s \in R_p} B_s (\mathbf{x}_s + \mathbf{y}_s) \leq B_p^U D_p \quad (3)$$

$$\sum_{s: B_s \in R_p} B_s (\mathbf{x}_s + \mathbf{y}_s) \geq B_p^L D_p \quad (4)$$

$$\sum_{s: B_s \in R_p} (O_s \mathbf{x}_s + O_s^{\text{RP}} \mathbf{y}_s) \leq O_p^U D_p \quad (5)$$

$$(\mathbf{x}_s + \mathbf{y}_s) \leq L_s \quad \{s : B_s \in R_p\} \quad (6)$$

$$\mathbf{x}_s, \mathbf{y}_s \geq 0.$$

### 3.1.2 The objective function

The objective function (1) is set up to minimise the overall costs associated with the operations as follows.

The per tonne cost,  $C_s$ , of unblended ore in the stakes is assumed to be equivalent to its value, which is related to its relative scarcity or abundance, and hence to its usefulness in creating blended talc. For the purposes of this preliminary model, we assume that:

1. The value of unblended ore is guided by the value of blended talc, that is, the final product. The value of blended talc is primarily dependent on the most important grade criterion, which is brightness.
2. Talc with a low primary grade is abundant and WMC wants to utilise as much of this low grade as possible during the blending procedure.

We adopted the following simple linear cost function based on these assumptions.

$$C_s = f(B_s - k) \quad \{s : B_s \in R_p\} \quad (7)$$

where  $B_s$  is the brightness of the talc in stake  $s$ , and  $f$  and  $k$  are constants.

### 3.1.3 The constraint equations

#### *Demand*

The constraint given by (2) states that the total amount of talc taken from the stakes, plus the amount repicked, must equal the demand for the product  $p$ .

#### *Brightness blending*

Constraint (3) states that the weighted average brightness of the talc taken directly from the stakes and the talc that is repicked must be less than the upper limit of the brightness level for product  $p$ .

Constraint (4) states that the brightness of the blended talc must be greater than or equal to the lower limit of the brightness level for product  $p$ .

In the constraints given by (3) and (4) we assume that the brightness of any repicked material is the same as that of the original material in the stake.

#### *Calcium oxide blending*

Constraint (5) states that the weighted average CaO level of the talc taken directly from the stakes and the talc that is repicked must be less than the upper CaO limit for product  $p$ .

#### *Stake inventory levels*

The set of constraints in (6) state that the total amount of talc taken from each stake cannot exceed the stake level.

## 3.2 Variations on the basic single order model

The basic model described above can be modified to incorporate practical operational constraints that may be required by Western Mining Corporation. We discuss some possible modifications in Sections 3.2.1–3.2.3. Although they are treated separately, the various modifications could be combined in different models as required.

### 3.2.1 Equipment movement costs

The various stakes are located in four main stockpile areas. When blending to fill a customer order, it is more economic to choose ore from a few stockpiles



rather than many since this will reduce the need to move equipment during loading operations.

We can incorporate this factor into the model by nominating one stockpile area to be a ‘home’ stockpile area. Ore taken from stakes in the other stockpile areas then incurs an additional per tonne transport cost  $C_T$ . This cost penalty makes it less attractive for the optimiser to choose from stakes in stockpile areas other than the home stockpile area.

If stockpile area  $A$  is the home stockpile area, then the cost function in (7) becomes

$$C_s = \begin{cases} f(B_s - k) & \{s : s \in A, B_s \in R_p\} \\ f(B_s - k) + C_T & \{s : s \notin A, B_s \in R_p\}. \end{cases} \quad (8)$$

With the cost function of (8), the model would be solved in turn with each of the four stockpile areas as the home stockpile area. The most favourable solution would then be used.

An alternative approach to restricting the use of stakes from different areas is to introduce binary variables,  $b_A$ , representing decisions on whether to use stockpile area  $A$ . We then rewrite the constraints in (6) as

$$(x_s + y_s) \leq L_s b_A \quad \{s : s \in A, B_s \in R_p\}, \{A : A \in \mathcal{H}\} \quad (9)$$

and introduce the constraint

$$\sum_{A:A \in \mathcal{H}} b_A \leq N \quad (10)$$

where  $N$  is the maximum number of stockpile areas to be used.

### 3.2.2 Blending using all or none of a stake

The solutions to the models described above will usually involve the partial usage of different stakes. If we want to avoid this partial usage, then we can use the binary variables  $b_s^x$  and  $b_s^y$  to represent decisions as to which complete stakes to use, and introduce the constraints

$$\begin{aligned} x_s &= b_s^x L_s \\ y_s &= b_s^y L_s \end{aligned} \quad \{s : B_s \in R_p\}. \quad (11)$$

The constraints in (11) specify that we either use all of a stake for direct blending of the final product, or pick all of a stake before blending it into the final product.

If we introduce these changes, then, in order to ensure that demand is satisfied, we must relax the demand constraints in (2) so that they become inequalities, namely

$$\sum_{s: B_s \in R_p} (x_s + y_s) \geq D_p. \quad (12)$$

Writing the demand constraints as in (12) means that, in the blending constraints (3), (4) and (5), we can no longer use  $D_p$  to represent the total amount of talc mined. Instead, we must use the left hand side of (12) explicitly. For example, the upper brightness limit constraints of (3) should be changed to

$$\sum_{s: B_s \in R_p} (B_p^U - B_s)(x_s + y_s) \geq 0 \quad (13)$$

with similar changes to (4) and (5).

### 3.2.3 Incorporating truck carrying capacities

If required, we can set the amounts taken from the stakes to be multiples of the capacity of the trucks used for transporting the talc by introducing integer variables  $j_s^x$  and  $j_s^y$  to represent the number of truck loads of material taken from the stakes. We then add the following constraints

$$\begin{aligned} x_s &= j_s^x T_C \\ y_s &= j_s^y T_C \end{aligned} \quad \{s : B_s \in R_p\} \quad (14)$$

where  $T_C$  is the carrying capacity (tonnes) of the trucks used to move the talc.

These constraints specify that the amounts taken from the stakes for blending or repicking are expressed in terms of a specified number of truck journeys.

Again, in order to ensure that demand is met, we need to use the demand constraints shown in (12), and to modify the brightness and CaO blending constraints as indicated in (13).

### 3.3 Multi-product multi-period model

This model is an extension of the single order model described in Sections 3.1 and 3.2. In the multi-product multi-period model we attempt to determine the optimal use of the stakes in satisfying the orders for all the products ( $p$ ) over a period of months ( $t$ ). We also include the fact that talc will be mined during the planning period. A series of typical orders is shown in Table 2 on page 146.

The variables and parameters used in this model are similar to those for the single order model with the following changes and additions.

#### Variables

- $x_{spt}$  Tonnes of stake  $s$  used directly for blending product  $p$  in period  $t$ .  
 $y_{spt}$  Tonnes of stake  $s$  repicked to achieve grade specifications for product  $p$  in period  $t$ .  
 $m_{pt}$  Tonnes mined to satisfy the demand for product  $p$  in period  $t$ .  
 $l_{st}$  Level of stake  $s$  at the beginning of period  $t$ .

#### Parameters and costs

- $P$  Number of products  $p$ .  
 $T$  Number of time periods  $t$  in the planning horizon.  
 $D_{pt}$  Demand for product  $p$  in period  $t$ .  
 $B_p^{\text{Md}}$  Median brightness level for product  $p$ .  
 $C_E$  Per tonne production cost of talc.  
 $C_M$  Per tonne cost of mining additional talc to meet demand.

The value of  $B_p^{\text{Md}}$  is calculated from the brightness limits as  $(B_p^{\text{U}} + B_p^{\text{L}})/2$ .

#### 3.3.1 The basic multi-product multi-period LP model

The LP model is formulated as follows.

$$\begin{aligned} \text{Minimise: } & \sum_{s: B_s \in R_p} \sum_{p=1}^P \sum_{t=1}^T C_E x_{spt} + \sum_{s: B_s \in R_p} \sum_{p=1}^P \sum_{t=1}^T (C_E + C_{RP}) y_{spt} \\ & + \sum_{p=1}^P \sum_{t=1}^T (C_M - kt) m_{pt} \end{aligned} \quad (15)$$

Subject to:

$$\sum_{s: B_s \in R_p} (x_{spt} + y_{spt}) + m_{pt} = D_{pt} \quad \begin{array}{l} p = 1 \dots P \\ t = 1 \dots T \end{array} \quad (16)$$

$$\sum_{s: B_s \in R_p} B_s (x_{spt} + y_{spt}) + B_p^{\text{Md}} m_{pt} \leq B_p^{\text{U}} D_{pt} \quad \begin{array}{l} p = 1 \dots P \\ t = 1 \dots T \end{array} \quad (17)$$

$$\sum_{s: B_s \in R_p} B_s(x_{spt} + y_{spt}) + B_p^{\text{Md}} m_{pt} \geq B_p^{\text{L}} D_{pt} \quad \begin{matrix} p = 1 \dots P \\ t = 1 \dots T \end{matrix} \quad (18)$$

$$\sum_{s: B_s \in R_p} (O_s x_{spt} + O_s^{\text{RP}} y_{spt}) + O_p^{\text{U}} m_{pt} \leq O_p^{\text{U}} D_{pt} \quad \begin{matrix} p = 1 \dots P \\ t = 1 \dots T \end{matrix} \quad (19)$$

$$l_{s1} = L_s \quad \{s : B_s \in R_p\} \quad (20)$$

$$l_{s(t+1)} = l_{st} - \sum_{p=1}^P (x_{spt} + y_{spt}) \quad \begin{matrix} \{s : B_s \in R_p\} \\ t = 1 \dots (T-1) \end{matrix} \quad (21)$$

$$\sum_{p=1}^P (x_{spT} + y_{spT}) \leq l_{sT} \quad \{s : B_s \in R_p\} \quad (22)$$

$$x_{spt}, y_{spt}, m_{pt}, l_{st} \geq 0.$$

### 3.3.2 The objective function

The objective function (15) is set up to minimise the overall costs associated with the operations as follows. The first expression represents the total cost associated with the stake material used for direct blending of the products. The second expression represents the total cost associated with stake material that needs to be repicked in order to meet product specifications. The third expression represents the cost of mining additional talc that may be needed to satisfy the product demand.

Note that the coefficient,  $(C_M - kt)$ , of the last expression includes a future discount factor. The term  $-kt$  is used to reduce the mining costs in later periods. This has the effect of discouraging extra mining, in a relative sense, during the early time periods.

### 3.3.3 The constraint equations

#### Demand

The constraints given by (16) state that, for product  $p$  in time period  $t$ , the total amount of product taken from the stakes, plus the amount repicked, plus the amount mined, must equal the demand.

#### Brightness blending

The constraints in (17) state that the weighted average brightness of the talc taken directly from the stakes, the talc that is repicked, and the talc that must

be mined to satisfy product demand, must be less than the upper limit of the brightness level for product  $p$  in time period  $t$ .

The constraints in (18) state that the brightness of the blended talc must be greater than or equal to the lower limit of the brightness level for product  $p$  in time period  $t$ .

For the constraints given by (17) and (18), we assume that if we need to mine additional material, because we cannot satisfy product demand from the existing stockpiles, then the material can be mined at a brightness level equal to the average brightness,  $B_p^{Md}$ , of product  $p$ . We also assume that the brightness of any repicked material is the same as that of the original material in the stake.

#### *Calcium oxide blending*

The constraints in (19) state that the weighted average CaO level of the talc taken directly from the stakes, the talc that is repicked, and the talc that must be mined to satisfy product demand, must be less than the upper CaO limit for product  $p$  in time period  $t$ .

We assume that if we need to mine additional material, then it is mined at a CaO level equal to the upper CaO limit for product  $p$ . In practice, additional talc would be mined and its grade determined. If the grade was within specification, then it would be used in the product. Otherwise it could be repicked to reduce the level of contaminants.

#### *Stake inventory levels*

These constraints are used to track the stake levels in the different stockpile areas.

The constraints in (20) set the initial stake levels in period 1.

The constraints in (21) are used to manage the stake levels in subsequent periods. These equations state that the stake level at the start of the second and subsequent periods is equal to the stake level at the start of the previous period less the amounts taken for direct blending and repicking in the previous period.

The constraints in (22) state that in the final period,  $T$ , the total amounts used for blending and repicking cannot exceed the stake levels. We do not need equivalent constraints for periods  $t = 1 \dots (T - 1)$  because they are implicit in the constraint set given by (21).

### 3.4 Variations on the multi-product multi-period model

As with the single order model, we can modify this basic multi-product multi-period model to accommodate different operating conditions.

In the multi-product multi-period model we use specified production costs in the objective function. These costs could be set up as in the objective function coefficients given in (8) for the single order model, to penalise the use of multiple stockpile areas when choosing stakes for blending.

Similarly, we can restrict the number of stockpile areas used for blending in time period  $t$  by introducing binary variables  $b_{At}$  representing decisions on whether to use stockpile area  $A$  in period  $t$ . We then add the sets of constraints:

$$\sum_{p=1}^P (x_{spt} + y_{spt}) \leq L_s b_{At} \quad \{s : s \in A, B_s \in R_p\}, \{A : A \in \mathcal{H}\} \quad (23)$$

$$\sum_{A:A \in \mathcal{H}} b_{At} \leq N_t \quad (24)$$

where  $N_t$  is the maximum number of stockpile areas to be used in time period  $t$ .

We can also introduce constraints specifying that the amounts of talc taken from the stakes must be some number of truck journeys.

During the MISG additional constraints, which limited the total amount of product that could be mined in a given time period, were also investigated. The effects of these constraints could not be tested due to lack of data.

## 4. Implementing the models

The models were tested using data supplied by Western Mining Corporation. Examples of typical data are shown in Tables 1–3.

The solver in Microsoft Excel was used to prototype the single order model. Excel can be used to create optimisation models that are simple to operate and the spreadsheet format provides output that is easy to understand. However, Excel has to sacrifice speed of operation for ease of use. While solution times for the LP formulation were satisfactory, Excel took several hours to obtain solutions to variants of the single order model containing integer variables.

All the variants of the single order and multi-product multi-period models were implemented using the GAMS modelling system, with OSL as the solver.

The OSL optimiser provided solutions to all the different single order models in less than one minute.

For the multi-product multi-period model, solving a problem with around 150 stakes in 4 stockpile areas, with demands specified over a full 12 month period, required about 35 minutes (around 45 minutes for the integer versions) on a 486 PC. However, because there are only about 3 months of products in the stakes at any given time, and because of the future discounting of the costs for mining additional product, the solutions for these 12 month models utilise all the stake material in the first 3 to 4 months. For the remaining months the solution simply contains amounts to be mined that balance the (already known) demands.

Testing the multi-product multi-period model with a look ahead of 4 months gave solutions that were essentially identical to the 12 month model. It is likely that the differences in the solution variables between the 4 month and 12 month models are due primarily to degeneracy in the models. Using a 4 month model, solutions were obtained in around 5–10 minutes.

Consequently, we feel that a 12 month planning horizon may be excessive, given likely changes to demand in the longer term, but nevertheless it may be appropriate to use a planning horizon which is sufficiently long to contain demands that just exceed the current capacity of the stakes.

Results from the different models were provided to Western Mining Corporation for comparison with current operating policies.

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