Risk and Return Performance Attribution for Cross Border Investment Portfolio

David Lamper

1 Questions Posed

Should currency be treated as a separate asset class? If currency hedging adds value to a portfolio, how should this be done?

2 Portfolio construction

Consider portfolio of domestic and foreign assets:

$$E[R] = w_d E[R_d] + w_f(E[R_f] + E[R_c])$$

where $E[R]$: Portfolio return, $E[R_d]$: Domestic asset return, $E[R_f]$: Foreign asset return, $E[R_c]$: Currency return , w_d : proportion in domestic assets, w_f : proportion in foreign assets and $w_d + w_f = 1$.

The addition of foreign asset exposure affects both the return and risk of a portfolio. Consider hedging foreign currency exposure using forward contracts: Domestic Assets; Foreign Assets; Hedge.

3 Currency hedge

Let S denote the spot foreign exchange rate and F denote forward exchange rate. Conversion agreed to now, but transaction takes place at some point in the future

Forward contract return should be

$$\frac{S(T)F}{S(t)} = R_c f$$

where f is the forward premium (or discount).

Combine forward contract with foreign asset, we have

$$(E[R_f] + E[R_c]) + h(E[R_c]f)$$

where h is the fraction of asset exposure hedged.

Rearranging it we obtain

$$(E[R_f] + f) + H(E[R_c] - f)$$

where H is the currency exposure ratio.

4 Portfolio return

Include currency hedge:

$$E[R] = w_d E[R_d] + w_f(E[R_f] + f) + H(E[R_c] - f)$$

with constraints $w_d + w_f = 1; 0 \leq H \leq w_f$
e.g. fully hedged ($H = 0$):

$$E[R] = w_d E[R_d] + w_f(E[Rf] + f)$$

5 Benchmark portfolios

Performance is often related to a benchmark portfolio. Risk and return should be measured relative to benchmark. Managers generate excess returns by deviating from benchmark.

Excess return :

$$E[\Delta R] = E[R] - E[R_{benchmark}]$$

$$E[\Delta R] = \Delta w_d E[R_d] + \Delta w_f(E[R_f] + f) + \Delta H(E[R_c] - f)$$

6 Risk

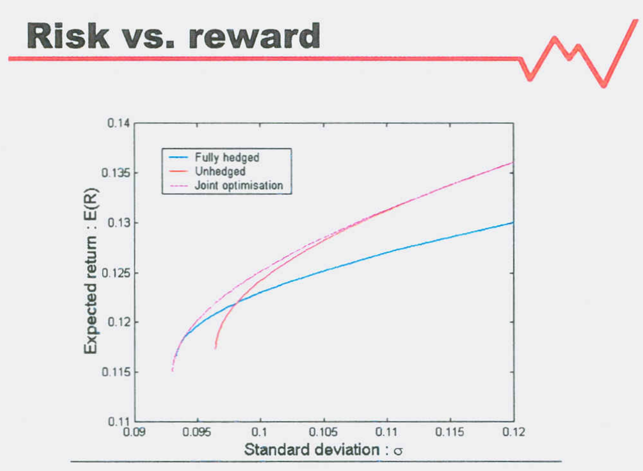
Performance of financial asset cannot be measured by the increase in capital alone, but also by risk incurred during time required to achieve this return.

Measures of risk include: Standard deviation σ, value at Risk (VaR), relative value at Risk (ReVaR), tracking error and many more .

7 Portfolio theory

Combination of portfolios A and B we can see

8 Risk vs.reward



9 Sample Parameters

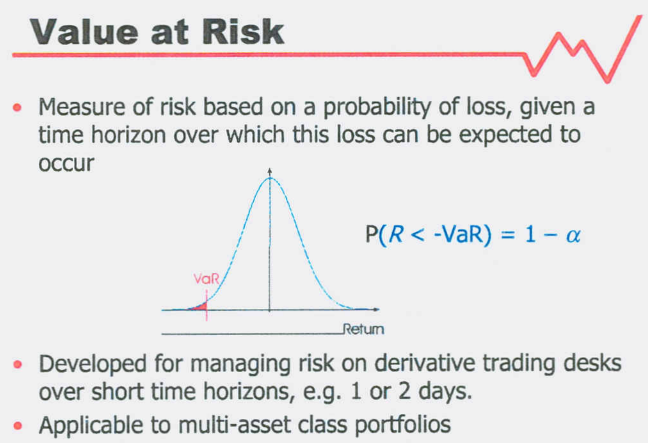
	Domestic	Foreign	Currency
Expected Return	$E[R_d] = 11$ %	$E[R_f] = 13$ %	$E[R_c] = 2$ %
Standard Deviation	$\sigma_d = 10$ %	$\sigma_f = 12$ %	$\sigma_c = 8$ %

Correlations:

$$\rho_{df} = 0.5,\ \rho_{dc} = 0.0,\ \rho_{fc} = 0.2$$

10 Value at Risk

Measure of risk based on a probability of loss, given a time horizon over which this loss can be expected to occur



Developed for managing risk on derivative trading desks over short time horizons, e.g. 1 or 2 days. Applicable to multi-asset class portfolios.

11 Long term VaR

Calculating VaR over long time horizon, e.g. 3 months, requires estimation of mean m and covariances S of assets within portfolio.

Incorporate hedge by treating as just another investment within portfolio.

Assume Normally distributed:

$$R = \sum w_i r_i$$

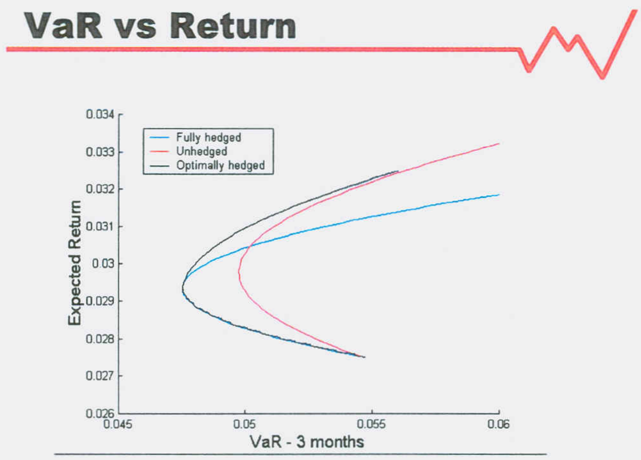
$$r_i \sim N(\mu_i, \sigma_i)$$

then

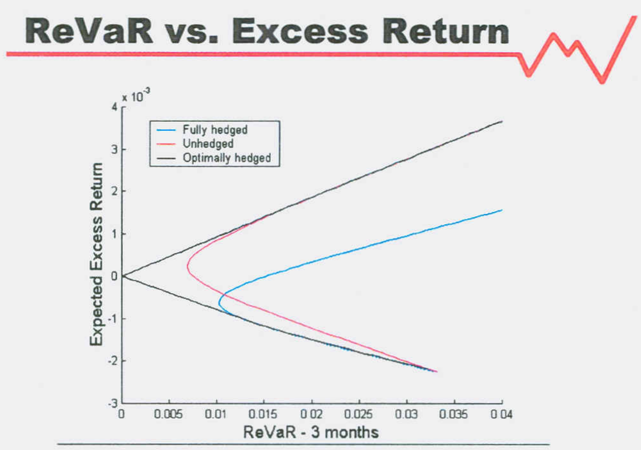
$$R \sim N(\mu w, (w^T \sum w)^{0.5})$$

Portfolio composition likely to change during time period, risk may be overstated.

12 VaR vs Return



13 ReVaR vs. Excess Return

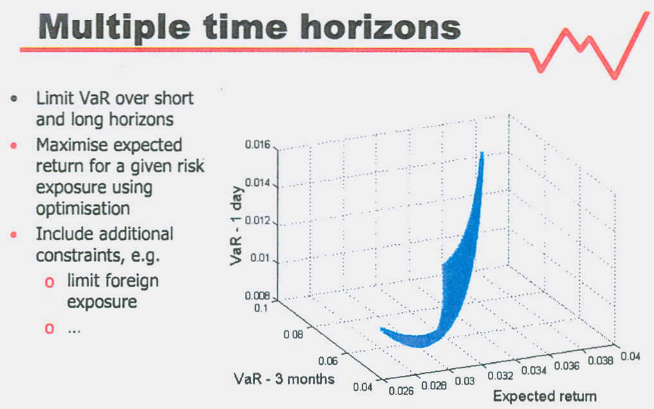


14 Multiple time horizons

Limit VaR over short and long horizons.

Maximize expected return for a given risk exposure using optimization.

Include additional constraints, e.g. limit foreign exposure

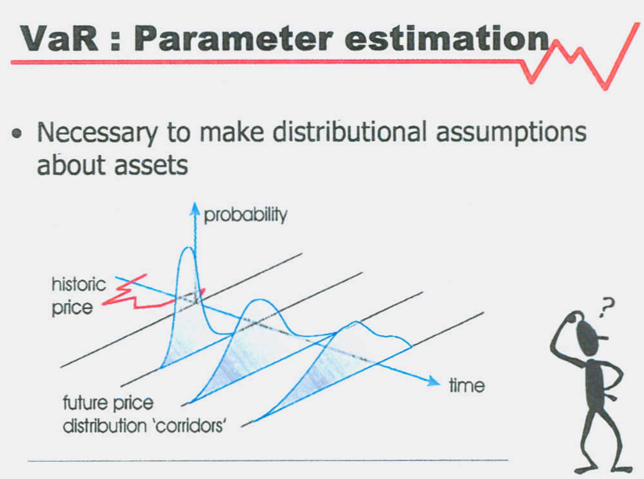


15 VaR : Implementation

Increasing the complexity: 1) Assume returns are Normally distributed, estimate parameters using historical data.2) Forecast parameters using market knowledge. 3)Incorporate more realistic asset model, e.g. GARCH, stochastic volatility, mean-reverting exchange rates, unstable correlations. 1) & 2) permit analytic expression for VaR. 3) requires simulation, but could improve accuracy. Backtesting on historical data should be performed. Can reduce size of correlation matrix using risk factors.

16 VaR : Parameter estimation

Necessary to make distributional assumptions about assets.



17 Summary

VaR can be extended to longer-term horizons, but parameters need to be estimated.

Currency hedging can improve return for a given risk exposure.

Hedge calculated during portfolio optimization, should not be treated as separate problem