Effect of distributed energy systems on the electricity grid

Problem presented by

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Ecotricity

Executive Summary

A feasibility study is being carried out at Ecotricity into a distributed energy storage system comprising Energy stores (batteries) placed at consumer level (in customer's homes). The aim is to flatten consumer demand and make better use of home-based generation. The Study Group considered the mechanism of connecting batteries to the local distribution system, the ability to meet engineering requirements for the standard of the connection, and the potential impact of large numbers of such connections on stability of the local distribution network. Network and (DC-AC) invertor models were used to examine network connection transients. A statistical model was proposed to estimate the distribution of key electrical parameters to determine the likelihood of engineering standards being exceeded. The Study Group also considered stochastic methods of modelling wind speed, to better understand the requirements for battery energy storage as a complement to wind power.

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Contents

1 Introduction

1.1 Background and scope

(1.1.1) Ecotricity has been concerned with building wind turbines and connecting them to the grid. It has built 15 wind parks across the UK, including a wind turbine on the outskirts of Cardiff at G24i (a solar panel manufacturer).

(1.1.2) Ecotricity wishes to develop a distributed energy storage system, based on the use of a black box to store electricity at consumer level. Consumer demand and wind-generated energy have peaks and troughs that do not match. Currently, Ecotricity sells electricity at low value during high wind generation and buys it at high value to fill the gaps; as a result, peak demand is met by CO_2 producing power generation and Ecotricity is unable to maximise its profits.

(1.1.3) A distributed energy storage system could flatten the demand curve and sell power on demand, enabling better value for Ecotricity. Flattening the grid/consumer demand profile would reduce the need for CO_2 producing power capacity, furthering Ecotricity's ethical stance and supporting the UKs green agenda, and possibly even help it to cope with the loss of the 8GW of coal fired power stations that are earmarked for closure by 2015.

(1.1.4) Ecotricity will seek the approval of the industry regulator and others before introducing energy exporting black boxes into the market (and into customer's homes). It is currently carrying out a feasibility study, in which a number of questions need to be answered, including the following:

- Assuming technology is available to store then distribute energy locally, what is the effect on the stability of the National Grid of introducing hundreds of thou- sands of black box energy stores?
- The AC output from all the large generators currently must to be synchronised, in terms of peak voltage, frequency and phase angle. Are the current standards (G83) for Grid Tied Inverters (that convert DC into AC) sufficient to ensure the introduction of an unprecedented number of energy stores or micro generators onto the grid will not affect its stability?
- What is the effect of different network topologies and can the micro generators interact?

(1.1.5) Ecotricity also posed further questions for consideration by the Study Group:

- What are the advantages of introducing many small energy stores as opposed to fewer larger ones?
- Where is best to locate them (near demand or generation)?

- Can existing simple models of lead acid batteries (e.g. Peukert's law) be extended to describe lithium based cells?
- Effect on the market of who will Ecotricity compete with?
- How can wind generated power be modelled?
- How do these questions relate to the simulation developed by the Department for Environment and Climate Change?

1.2 Overview of AC power systems

(1.2.1) The modelling approach suggested by Ecotricity, comprises a complex AC power-flow network of nodes (sources and sinks) and edges (transmission and distribution lines). In this framework, electro-magnetic transients are neglected, and so the dynamics of power-flows in the network arise from the dynamics of the sources (generators) and sinks (demands), and from changes in the network topology (as, for example, caused by opening circuit breakers). When at steady-state, whole network runs at a uniform frequency (50Hz); however, owing to impedance in the transmission lines, there are spatial phase shifts between the nodes which vary according to the power-flow between them. This network modelling formalism is well-established, and is used by electricity network companies, such as National Grid, for modelling events on timescales $\tilde{>}$ 1 second. (Modelling of electromagnetic transients on much shorter time scales is also done, but is not relevant here.)

(1.2.2) The network is tiered into levels of different voltages (from 475 KV to 240 V) with are linked by transformers at sub-stations. The highest level network (to which centralised generation is connected) is called the grid system, and the low level networks (to which consumers are connected) are referred to as distribution systems.

(1.2.3) There are engineering standards for the operation of the system at all levels, which relate to frequency and voltage control, and there are connection standards that must be met by users (generation and demand). These standards (such as G83) maintain the quality of supply and the operational stability of the transmission and distribution systems under sudden changes.

1.3 The effects of distributed generation

(1.3.1) There is rapidly growing research and literature on the effects of increasing levels of distributed generation upon the quality and stability of electricity supply. Examples that were examined by the Study Group include [1, 4, 5, 6], which are primarily concerned with network synchronisation and transient stability. They illustrate the use of complex (real(active) and imaginary (reactive)) power-flow models for networks and of dynamic models for the attached generators and loads. In a theoretical analysis, Dörfer and Buller [5] emphasise the importance of network topology for synchronisation and stability, and draw attention to a consensus protocol that can drive synchronisation. At the microgrid level, Hiskens and Fleming [4] illustrate the effects of battery (DC-AC) inverter dynamics upon distribution network dynamics following a sudden change in network topology (i.e. opening a circuit breaker). This latter reference was closely examined by the Study Group, owing to its detailed modelling of a constant voltage inverter for use in connecting a battery to an AC network. Other related literature, considers the use of such invertors in connecting domestic solar power systems to a local distribution network.

(1.3.2) The primary activity of the Study Group was to understand the collective effects of household generation connected through, inverters. (Newman et al. [1] examine collective effects of distributed generation by running many dynamic power-flow simulations, but not at the household level.) The Study Group investigated two approaches (i) dynamic modelling of a group of battery holding households connected to a local distribution network (ii) statistical analysis of the collective effects of a large number of such connections.

(1.3.3) The Study Group also considered stochastic methods for modelling wind speed, with a view to better understanding the capacity requirements for and demands upon battery energy storage as a complement to wind power.

2 Modelling the synchronisation of independent generators

2.1 Simple model for synchronisation

(2.1.1) In considering the uncoordinated connection of large number of independent household generators to a distribution network the potential relevance of consensus driving mechanisms was considered. These might have beneficial or detrimental effects. The following model was developed in relation to an ensemble of orchestral players following a score without a conductor.

(2.1.2) Consider n musicians playing together with no conductor, so they have to synchronize by listening to each other. In a simple model suppose player i is at position $\phi_i(t)$ in the score, he has constant natural speed Ω_i, and he modifies his actual speed according to how far his position deviates from the consensus position

$$\bar{\phi} = \sum_i w_i \phi_i \qquad (w_i > 0, \sum_i w_i = 1) \tag{1}$$

which is a weighted mean of the positions of all the players. In a linear model, he plays at speed

$$\omega_i = d\phi_i/dt = \Omega_i + \alpha_i(\bar{\phi} - \phi_i), \tag{2}$$

with $\alpha_i > 0$ constant.

(2.1.3) This model has a common speed $\bar{\omega}$ that the players can all play at, $\bar{\omega}$ is a global attractor (*i.e.*all players converge to it from any starting conditions), and when converged each player has a characteristic position offset from the consensus.

(2.1.4) To find the common speed $\bar{\omega}$, we note that

$$\sum_i \frac{w_i}{\alpha_i}\omega_i = \sum_i \frac{w_i}{\alpha_i}\Omega_i + \sum_i w_i(\bar{\phi} - \phi_i), \tag{3}$$

and the last sum is identically 0, so

$$\bar{\omega} = \left(\sum_i w_i\Omega_i/\alpha_i\right) \Big/ \left(\sum_i w_i/\alpha_i\right) \tag{4}$$

is the only possible common speed: the natural speeds Ω_i are averaged with weights w_i/α_i.

(2.1.5) To show the global stability property, consider the quantity

$$V = \frac{1}{2}\sum_i w_i(\omega_i - \bar{\omega})^2/\alpha_i^2. \tag{5}$$

The time derivative of this can be found using $d\omega_i/dt = \alpha_i(d\bar{\phi}/dt - \omega_i)$, and is

$$\begin{aligned} \frac{dV}{dt} &= \sum_i \frac{w_i(\omega_i - \bar{\omega})(d\bar{\phi}/dt - \omega_i)}{\alpha_i} && (6)\\ &= -\sum w_i(\omega_i - \bar{\omega})^2/\alpha_i && (7)\\ &\leq -2V\alpha_{\min}. && (8) \end{aligned}$$

Hence V is exponentially decaying with $V(t) \leq V(0)\exp(-2\alpha_{\min}t)$, and so each $\omega_i \to \bar{\omega}$ as $t \to \infty$.

(2.1.6) This convergence of the speeds to $\bar{\omega}$ shows that the position offsets also converge,

$$\phi_i - \bar{\phi} = \frac{\Omega_i - \omega_i}{\alpha_i} \to \frac{\Omega_i - \bar{\omega}}{\alpha_i}. \tag{9}$$

So the players whose natural speed is above the common speed $\bar{\omega}$ will converge to an offset where they are slightly ahead of the consensus, and those whose natural speed is slower will converge to a position slightly behind.

(2.1.7) The analogue of this in the electrical context is that we think of $\phi_i(t)$ as analogous to the phase of inverter i, so that $\omega_i = d\phi_i/dt$ is its instantaneous frequency. The phase of the network is like the consensus phase $\bar{\phi}$. Then if the inverters are transmitting power to or from the network, that is represented by larger or small values of Ω_i, and a corresponding phase difference $\phi_i - \bar{\phi}$ between the inverter and the grid. The equation (2) would need to be replaced by a system representing the phase-locking mechanisms of the inverters, and if that is more complicated than the simple proportional control considered here then the stability and instability properties may be more complex. When it is desired to keep the phase differences $|\phi_i - \bar{\phi}|$ small (so that there is no cycle-skipping in the phase-locked loops, and so that the sines of these phase differences can be approximated by the angles themselves), that would require having large α_i in this model. But in a model with more complex dynamics, taking the gain α_i too large may result in instability. So the usual control theory trade-offs can be expected to arise, but with the added condition that we want the global properties of the system to be maintained for all n.

(2.1.8) The above consensus model may be related to the consensus protocol given in [5] for a system of n autonomous agents each characterised by a state x_i. Here consensus is defined as $x_i(t) - x_j(t) \to 0$ as $t \to i\infty$. Given an adjacency matrix A describing their interaction, the following consensus protocol

$$\dot{x}_i = \sum\nolimits_i^n a_i j(x_i - x_j), i \in \{1, ..., n\} \tag{10}$$

is a linear continuous time algorithm to achieve consensus. This protocol makes explicit the dependency on the topology of the connections between the agents.

2.2 Other work on collections of coupled oscillators

(2.2.1) Potentially relevant to the analysis of small generator synchrony, is research on biological oscillators [2].

(2.2.2) Along the same lines, there is the Fermi-Pasta-Ulam work on a lattice of nonlinear oscillators coupled by nearest-neighbour interactions [3], or more recent work on coupled Lorenz systems. Both show complicated quasi-periodic behaviour.

2.3 Statistical modelling of an ensemble of households

(2.3.1) Engineering standards place limits (±18 degrees) on the variation in phase between nodes in distribution network. A potentially useful approach is to estimate distribution of the collective effect of a large number of household connections with randomly distributed phases.

(2.3.2) Assume that a household i may be represented as a voltage source V_i with phase ϕ_i (relative to the rest of the grid system) behind an impedance Z_i which is then connected to the distribution network supporting n such households. Let the remaining part of the network be represented by a single voltage V_0 with phase ϕ_0 behind an impedance Z_0. Let the whole distribution network be connected to the rest of the grid system through a sub-station at voltage V_s, which has angular frequency ω ($100\pi s^{-1}$). Then the power contribution ΔP of the distribution network to the whole system is:

$$\Delta P = \frac{V_s V_0}{Z_0} \exp\left(i\phi_0 + i\omega t\right) + \sum_i^n \left(\frac{V_s V_i}{Z_i} \exp\left(i\phi_i + i\omega t\right) \right) \tag{11}$$

(2.3.3) In this equation, we may work in the rotating frame of the grid and set $\omega = 0$. Suppose now that, the phases $\{\phi_i | i = 1, ..., n\}$, are normally distributed $\sim N(0, \sigma)$, and, moreover, that they are small, by virtue of satisfying the connection standards. Then we may write

$$\Delta P = \frac{V_s V_0}{Z_0}(1 + i\phi_0) + \sum_i^n \left(\frac{V_s V_i}{Z_i}(1 + i\phi_i) \right), \tag{12}$$

whence it follows from assuming the normality of the ϕ_i that $\Delta P \sim N(P_0, \sigma\sqrt(n))$. Thus, to a good approximation the normal distribution for ΔP may be used to estimate the increased likelihood that the complex power trip limits of the substation may be exceeded, and a circuit breaker opened.

(2.3.4) Such statistical analysis may help estimate the likelihood of significant random collective effects.

2.4 Transient modelling of an ensemble of households

(2.4.1) The connections of many households to a distribution network through (perhaps similar or even identical) DC-AC inverters which have higher than first order response characteristics, may systematically lead to coupled modes of behaviour in the network that grow and lead to connection standards and/or operating standards being exceeding. Investigation of the possibility, extent and prevention of such behaviours requires dynamic systems analysis. Existing work in this area includes network modelling and simulation e.g. [4], and theoretical work [5] on establishing algebraic conditions on the structure of a network, to secure its stability.

(2.4.2) The Study Group drew on [6, 4], in particular, with the aim of formulating a model and then simulating the behaviour of an ensembles of households connected to a distribution network. The overall aim would be to gain understanding of the transient behaviour of such systems and to investigate conditions under which unacceptable transients might occur, and to highlight ways in which they may be designed out.

(2.4.3) Two distinct aspects of modelling are required and must be integrated; one of the dynamics of the generations and demand systems connected to the network; and the other of the resulting power-flow through the network. These equations are typically assembled into a coupled system of differential-algebraic equations

$$\dot{x} = f(x, y) \tag{13}$$
$$0 = g(x, y), \tag{14}$$

in which x is a dynamic state variable, and y an algebraic variable of the network power-flow. See [6]. The numerical solution of such systems using computer codes is by now well-established. However, the Study group set out to investigate how to simulate a small network of households, some grid-tied to provide power to the network and others not so. This requires a network power-flow model and models of the dynamics elements attached to it - in particular DC-AC inverters.

(2.4.4) At each network bus (node) the voltage and phase are variables, and the buses may have load and generation attached. The active (real) and reactive (imaginary) power balance equations for each load (and generation represented as negative load) are:

$$\sum\nolimits_j V_k V_j b_i j \sin(\phi_k - \phi_j) = P_k \tag{15}$$
$$\sum\nolimits_j V_k V_j b_i j \cos(\phi_k - \phi_j) = -Q_k \tag{16}$$

in which $b_i j$ is the imaginary part of the admittance between nodes i and j. Here, $Y = Z^{-1}$ where Z is the impedance matrix. The real part of Y is usually ignored as network losses are $\tilde{2}$%.

(2.4.5) The main dynamical elements to be modelled are the inverters in the grid-tied households. The inverters provide phase-locking and active-power regulation control loops, with integral feedback, and so are potentially a source of oscillatory response and interaction between houses unless they are sufficiently damped. The question arises as to whether such sufficient damping would compromise or be consistent with the ability of the network to meet operational standards under certain events such as circuit breakers opening for external reasons. What are the consequences of independent or of uniformly instructed connections and exports of power from large groups of households?

(2.4.6) The Study Group took the inverter design in [4, 7] as a basis for its plan of investigation; the same references provide a worked example of transient behaviour in a microgrid with only two inverter-connected sources. The investigation was continued after the Study Group; the resulting analysis and conclusions are reported in the Appendix of this report.

3 Stochastic Modelling of Wind Speed Data

3.1 A short introduction

(3.1.1) Modern distribution systems integrated with renewable generation bring a number of challenges to the optimal operation and planning of the systems. For the system operation and planning, probabilistic load flow techniques can be employed to obtain a realistic evaluation of the system steady-state performance. Thus, the objective is to develop stochastic models of wind speed through a probabilistic approach.

3.2 Empirical results

(3.2.1) Let us first consider the empirical results. Figure 1 shows the time series plot of the wind speed data, Figure 2 depicts the histogram, and finally Figure 3 shows the autocorrelation function.

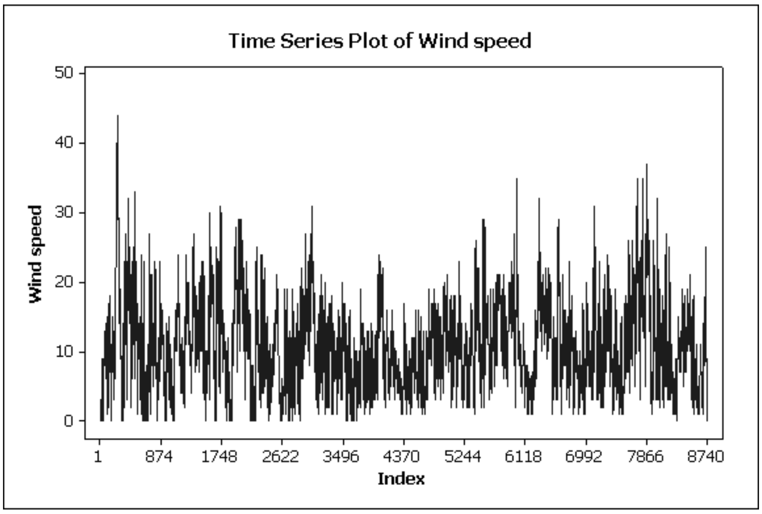

Figure 1: Time series plot the wind speed data.

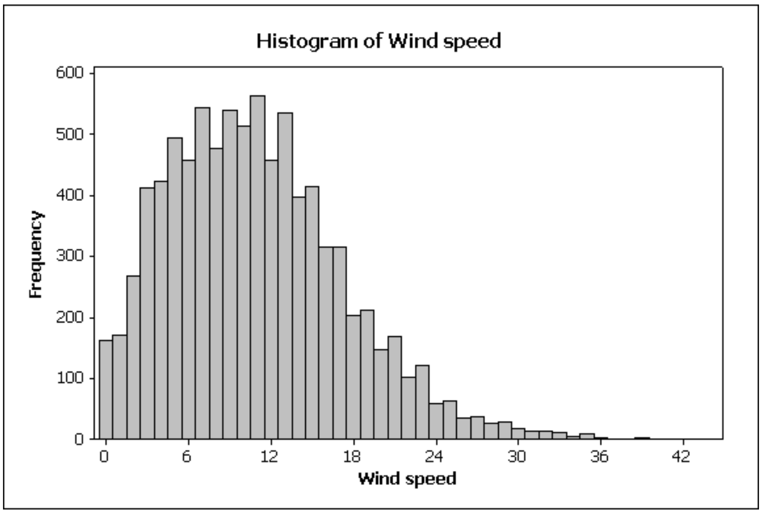

Figure 2: Histogram of the wind speed data.

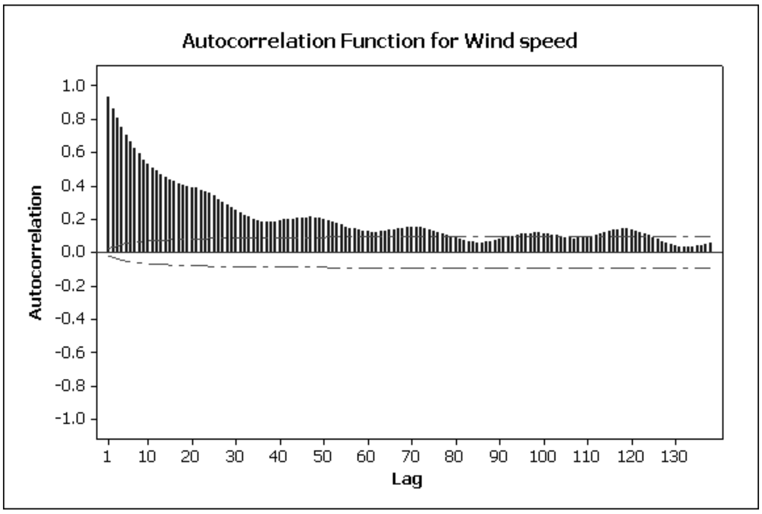

Figure 3: Auto-correlation function for the wind speed data.

3.3 Theoretical results

(3.3.1) Modern distribution systems integrated with renewable generation bring a number of challenges to the optimal operation and planning of the systems. For the system operation and planning, probabilistic load flow techniques can be employed to obtain a realistic evaluation of the system steady-state performance. Thus, the objective is to develop stochastic models of wind speed through a probabilistic approach.

(3.3.2) Problems: the questions of interest are: let $\xi(t), t \in R$, be a wind process/field, then

- Q1) what is the distribution of

$$P\left(\xi(t) < Z\right) = ?$$

- Q1) what is the distribution of random functionals:

$$|0 \le t \le T : \xi(t) < Z| = ?$$
$$|0 \le t \le T : \xi(t) > L| = ?$$
$$|0 \le t \le T : L < \xi(t) < Z| = ?$$

(3.3.3) The answers depend on the distributional properties of the wind field and on the correlation structures.

(3.3.4) It is well documented that the wind field has a marginal Rayleigh distribution, and given covariance function

$$B(t) = cov(\xi(0), \xi(t)),$$

which should be of different type, including short memory, long memory with different types of seasonalities.

(3.3.5) Time-dependent model: We need a model for a stationary process $\xi = \{\xi(t), t \in \mathbb{R}\}$, with Rayleigh distribution:

$$p(x) = \frac{x}{\sigma^2} \exp\left\{-\frac{x^2}{2\sigma^2}\right\}, x > 0,$$

cdf

$$P\left(\xi(t) < Z\right) = 1 - \exp\left\{-\frac{Z^2}{2\sigma^2}\right\}, Z > 0.$$

(3.3.6) Then our functionals take the forms:

$$S_T = |0 \le t \le T : \xi(t) < Z| = \int_0^T 1_{\xi(t) < Z} dt.$$
$$S_T = |0 \le t \le T : \xi(t) < Z| = \int_0^T 1_{L < \xi(t)} dt.$$
$$S_T = |0 \le t \le T : \xi(t) < Z| = \int_0^T 1_{L < \xi(t) < Z} dt.$$

a for the covariance structure $B(t) = cov(\xi(0), \xi(t))$, one can consider the following models:

(3.3.7) (A) short-range dependence

$$\int B(t)dt < \infty,$$

for example

$$B(t) = Ae^{-Bt}, t > 0, \text{or } B(t) = \frac{A}{(1 + Bt^2)^{\alpha/2}}, \alpha > 1,$$

(3.3.8) (B) long-range dependence:

$$\int B(t)dt = \infty,$$

for example

$$B(t) = \frac{A}{(1+Bt^2)^{\alpha/2}}, \alpha \in (0,1), \text{ or } B(t) = \frac{\cos(t\phi)}{(1+Bt^2)^{\alpha/2}}.$$

(3.3.9) i) One possible model could be

$$\xi(t) = \sqrt{Y_1^2(t) + Y_2^2(t)},$$

where $Y_j(t), j = 1, 2,$ are two independent Gaussian processes with marginal $N(0, \sigma^2)$ pdf's, and the covariance structure as $B_G(t)$, then covariance structure of $\xi(t)$ can be approximated by $B(t)$.
One can expect the following results: for the case (A):

$$\frac{S_T - T(1 - \exp\left\{-\frac{Z^2}{2\sigma^2}\right\})}{VarS_T} \approx N(0,1), T >> 1.$$

where $VarS_T$ is different for the cases (A) and (B).

(3.3.10) ii) Another model could be a mean reverting diffusion process with Rayleigh ergodic distribution:

$$dX_t = -\theta\left(X_t - \sigma\sqrt{\frac{\pi}{2}}\right)dt + \sqrt{v(X_t)}\,dB_t, \quad t \geq 0, \theta > 0,$$

where B_t is a Brownian motion, where the special choice of diffusion σ guarantees the Rayleigh ergodic distribution, that is

$$v(x) = \frac{2\theta \int_0^x (z - \sigma\sqrt{\frac{\pi}{2}})\frac{z}{\sigma^2}\exp\left\{-\frac{z^2}{2\sigma^2}\right\}dz}{\frac{x}{\sigma^2}\exp\left\{-\frac{x^2}{2\sigma^2}\right\}} = \frac{2}{x}\frac{\theta}{e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}}\int_0^x \left(z^2 e^{-\frac{1}{2}\frac{z^2}{\sigma^2}} - \frac{1}{2}z\sigma\sqrt{2\pi}e^{-\frac{1}{2}\frac{z^2}{\sigma^2}}\right)dz$$

(3.3.11) Spatiotemporal random fields: Propagation models of spatiotemporal random fields with possible long-range dependence and seasonalities in time and/or space.

(3.3.12) Let $\xi(x,t), x \in \mathbb{R}^2, t \in (0,\infty)$ denote a spatiotemporal random field with a covariance function homogeneous (and isotropic) in space and stationary in time

$$B(\|x-y\|, t-s) = Cov(\xi(x,t), \xi(y,s).$$

(3.3.13) It is not easy to specify covariance functions because these need to be non-negative definite. We plan to use the Gneiting (2002) theorem for generating spatiotemporal covariances: suppose $\varphi(u), u \geq 0$, is a completely monotone function, and let $\psi(u), u \geq 0$, be a function with a completely monotone derivative. Then

$$B(\|z\|, \tau) = \frac{\sigma^2}{\psi(\tau^2)^{d/2}} \varphi(\frac{\|z\|^2}{\psi(\tau^2)}), \sigma^2 \geq 0,$$

is a covariance function. One can use the following model:

$$\xi(t) = \sqrt{Z_1^2(x,t) + Z_2^2(x,t)}, x \in \mathbb{R}^2, t \in (0,\infty),$$

where $Z_j(x,t), j = 1,2$, are two independent Gaussian fields with marginal $N(0,\sigma^2)$ pdf's, and the given covariance structure.

We want it to have possibly long-range dependence in time or in space or in space and in time. We shall use for φ the Mittag-Leffler functions since they display a range of dependence structures and are complete monotone and have some additional nice properties.

(3.3.14) It is known that the generalized Mittag-Leffler function of negative real argument

$$E_{\alpha,\beta}(-u) = \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(-u)^k}{\Gamma(\alpha k + \beta)}, \ u \geq 0, \ \alpha, \beta > 0$$

is infinitely differentiable and completely monotone if and only if $0 < \alpha \leq 1$, $\beta \geq \alpha$.

(3.3.15) Note that $E_{\alpha,\beta}(0) = 1$. In particular, for $\alpha = 1, \beta = 1$, we have $E_{1,1}(-u) = e^{-u}$, $u \geq 0$, and for $0 < \alpha < 1$, $\beta \geq \alpha$

$$E_{\alpha,\beta}(-u) = \frac{\Gamma(\beta)}{|u|\Gamma(\beta-\alpha)} + O\left(\frac{1}{|u|^2}\right)$$

as $|u| \to \infty$. Thus the $E_{\alpha,1}(-u^\gamma)$ provides a smooth interpolation between the negative exponential function e^{-u} and the asymptotically power function $\frac{1}{|u|^\gamma \Gamma(1-\alpha)}$ for $|u| \to \infty$. This makes it a convenient tool for interpolating between short- and long-range dependence (SRD and LRD).

(3.3.16) The function

$$E_{\alpha,1}(-u^{\gamma}), 0 < \gamma < 1, 0 < \alpha \leq 1, u \geq 0$$

is also complete monotone, while the function

$$\psi(u) = (1 + au^{A})^{-\nu}, a > 0, 0 < A \leq 1, \nu > 0, u \geq 0$$

has a complete monotone derivative (as well as $\psi(u) = \log(au^{A}+b)/\log b, a > 0, b > 1, 0 < A \leq 1$, or $\psi(u) = (au^{A} + b)/(b(au^{A} + 1)), a > 0, 0 < b \leq 1, 0 < A \leq 1$). We will use these two functions in order to define covariance structure.

(3.3.17) Thus we can consider a class of covariances with short or long-range dependence in time and/or in space

$$C(\|z\|, \tau) = \frac{\sigma^2}{(a|\tau|^{2A} + 1)^{Bd/2}} E_{\alpha,1}(-\frac{\|z\|^{2\gamma}}{(a|\tau|^{2A} + 1)^{B\gamma}}), 0 < \gamma \leq 1, 0 < \alpha \leq 1,$$

$$a > 0, 0 < A \leq 1, 0 < B \leq 1.$$

or, using the Mittag-Leffler function

$$C(\|z\|, \tau) = \frac{\sigma^2}{(a|\tau|^{2A} + 1)^{Bd/2}} E_{\alpha,\beta}(-\frac{\|z\|^{2}}{(a|\tau|^{2A} + 1)^{B}}),$$

for

$$0 < \alpha \leq 1, \beta \geq \alpha, a > 0, 0 < A \leq 1.$$

(3.3.18) Observe that, if $\frac{\|z\|^2}{(a|\tau|^{2A}+1)^B} \to \infty$, we get for $0 < \alpha < 1$, the following asymptotic behavior for the covariance function ,

$$\frac{\sigma^2}{\Gamma(1-\alpha)} \frac{\sigma^2}{(a|\tau|^{2A} + 1)^{Bd/2-B\gamma}} \frac{1}{\|z\|^{2\gamma}}, 0 < \gamma \leq 1, 0 < \alpha < 1,$$

which provides an asymptotic separation of the time τ and space z variables.

(3.3.19) We have long-range dependence in space if $0 < \alpha < 1$, but also an exponential decay of covariance in space if $\alpha = 1$. We get a decrease in time as $|\tau|^{-2AB(\frac{d}{2}-\gamma)}$, and for $d \geq 2$ (d is the space dimension), we get long-range dependence in time, while for $d = 1$, long-range dependence only when $0 < \gamma < 1/2$.

A Appendix: Stability of inverters in power networks

This appendix was contributed after the Study Group by Chris Budd and Chris Hodge of the University of Bath.

A.1 Introduction

(A.1.1) Power supply networks have traditionally comprised large generating plant providing significant electrical power to many households, each of which was regarded as a passive (quasi-static) load on the system. However this situation has changed markedly in recent years, and there is much further change ahead, with the coming of the Smart Grid. In the new environment, households themselves become producers and storers of electricity through a variety of mechanisms such as solar cells, heat pumps and battery storage. Whilst relatively low in power (typically between 1kW and 10kW), the large number of such users, the intermittency of the power supplied by such users and the irregular geographical spread, all lead to a possibility of overload, voltage drop or destabilisation (particularly phase drifting) of the electricity supply network. This is compounded by the strong likelihood that in the future, domestic households will have electrical vehicles which will need to be charged overnight. The charging (about 3kW) per electrical vehicle will put a further load on the network.

(A.1.2) Power networks consist of a series of nodes (usually referred to as buses) connected by cables. Power is usually supplied to such a network by distant generators which have a large capacity to supply power. Typical users of this power are domestic housing, with an average household consuming between 400W and 1kW (or about 3500kWHours per year). A typical bus has associated with it a complex voltage $V = |V|e^{j\delta}$, a complex current I and a complex power $S = P + jQ$. The *real power* P is what is required to power the household and typically $P < 0$ to represent a power drain. The *reactive power* is Q. Power distribution networks usually try to maintain a supply which meets the required load demand of many houses as efficiently and stably as possible within the constraints of the system.

(A.1.3) However, in the Study Group, we were asked to consider the opposite problem in which the households become *net suppliers* of electricity to the grid. In practice this can arise from local energy sources such as solar panels, wind energy or heat pumps, or alternatively from batteries, or Solid Oxide Fuel Cells (SOFC) which are charged overnight. Such sources are typically suppliers of *Direct Current* (DC) at a low voltage (eg. 12V). To match to the grid this needs to be converted to Alternating Current (AC) and raised to the domestic supply voltage of 240V. This is achieved through an *inverter* device. The inverter also has to match the *phase* of the supplied voltage to that of the grid, so that the energy from the local source is delivered efficiently into the global network. Inverters have their own control systems (typically *phase-locked-loops*. However, they introduce new dynamics into the grid. Problems arise if this additional dynamics (particularly when there are a lot of households involved) can destabilise the operation of the grid as a whole. In this report we will look at the dynamics of such power generation systems and address the question of whether the system can become unstable due to the action of the inverter system attempting to synchronise the household supply with that of the grid, or whether the system has a degree of self-stabilisation and can synchronise itself.

A.2 The power load equations

(A.2.1) It is useful to review the basic equations of power flow in an electricity supply grid. Let a node be donated by i, with the complex voltage, current and power at the node given by V_i, I_i and S_i respectively. Typically S_i will be a combination of input powers S_i^I and output powers S_i^O so that $S_i = S_i^I - S_i^O$. We will assume that in general S_i^O is the normal household load of about 500W and that S_i^I will be the output of the inverter device.

(A.2.2) The power of the network is given by

$$S_i = V_i I_i^*, \quad S_i^* = V_i^* I_i \tag{17}$$

Typically the node i will be connected to a set of other nodes k via cables of impedance $z_{ik} = R_{ik} + jX_{ik}$ and conductance $y_{ik} = 1/z_{ik}$. Note that in a typical distribution network the cables are designed to be as loss free as possible. In such a case it is usual that $R \ll 1$ so that $z_{ik} = jX_{ik}$. The current flowing through this cable is then given by

$$I_{ik} = (V_i - V_k) y_{ik}, \tag{18}$$

so that the total current is given by

$$I_i = \sum_k (V_i - V_k) y_{ik} \equiv \sum_k Y_{ik} V_k \equiv (Y\mathbf{V})_i \tag{19}$$

where $\mathbf{V} = (V_1, V_2, \ldots)$ and the *admittance matrix* Y is defined by

$$Y_{ii} = \sum_k y_{ik}, \quad Y_{ik} = -y_{ik}, \quad i \neq k. \tag{20}$$

The *power flow equation* for each node is then given by

$$S_i^* = V_i^* (Y\mathbf{V})_i. \tag{21}$$

(A.2.3) If we suppose that the network has N nodes then this gives N-complex or $2N$-real equations. At each node/bus there are typically two unknowns. At a *slack bus* such as a generator, the complex voltage V_i is *known* and the power S_i has to be determined. At a *load bus* such as a household, the complex power is *known* and the complex voltage V has to be determined.

(A.2.4) In a static case, the system of quadratic equations (21) determine the operating state of the system. These are typically solved by iterative methods such as the Newton-Raphson method and its various derivatives. (Note that as the equations are quadratic it is possible to make extensive use of conjugate gradient techniques to speed up the calculations). There is quite a lot of software around which will do this, for example the (free) Matlab software `matpower`.

(A.2.5) NOTE: It is conventional in such codes to specify the voltage in terms of its *amplitude* $|V_i|$ and *phase* δ_i relative to a reference voltage.

A.3 The dynamics of inverters

(A.3.1) We now consider a typical household. In such a household the (AC) real power P_i will be the power P_i^S *generated* (by solar or other means) minus the power due a typical household load (of between 400W and 1kW) and the power (of about 3kW) required to charge an electrical vehicle. As a typical solar panel delivers DC, an inverter is needed both to generate the AC and also to match it to the grid. Accordingly, the power flow equations (21) need to be extended to include dynamical effects when inverters are included into the system. However, to quote a power engineer at Bath *"power inverters are a pain. They have little inertia and are unstable, so they have to be managed very carefully"*. (For the record, the University of Bath has an experimental large scale inverter in its library.) Unfortunately, our analysis implies that this received wisdom seemed to be correct!

(A.3.2) Synchronizing the inverter and grid AC voltage waveforms can be achieved using a phase-locked loop (PLL). Inverters act to supply AC to the grid at a voltage V_i and phase δ_i from the DC supply of the batteries via a pure impedance jX (typically a transformer) supplied at a voltage V_t and phase δ_t. The inverter seeks to regulate the active power P_{Gen} delivered to the grid from the batteries, and the terminal bus voltage magnitude V_i. This is achieved by controlling the modulation index m of the inverter as well as the inverter firing angle θ, which is equivalent to the phase of the synthesized voltage waveform. The phase δ_t of the inverter voltage must be established relative to a local reference signal. A phase-locked loop (PLL) is typically used in an inverter to provide this local reference. This leads to a series of ordinary differential-algebraic equations for the various unknowns.

(A.3.3) Suppose that P_{Gen} is the power supplied to the network, V_i is the network voltage and V_t is the voltage at the output of the inverter then

$$P_{gen} = \frac{|V_i||V_t|\sin(\delta_t - \delta_i)}{X} \tag{22}$$

Hence, the value of δ_t plays a huge role in determining the transmitted power. Assuming V_i and V_t remain relatively constant, regulation of P_{gen} can be achieved by controlling the angle difference $\delta_i - \delta_t$. If δ_t is not close to δ_i then we can see a large transient spike in the system during which a lot of power flows. This is a significantly destabilising factor in the coupled inverter/grid system. The inverter AC frequency is ω_p and this changes during the operation, leading to a change in δ_p and δ_t.

(A.3.4) When it is working well, the inverter steadily tracks the phase δ_t keeping it in synchronisation with the changing phase of the power grid. However, certain problems can arise from this approach (which can be analysed through the resulting differential equations):

(a) If the inverter is isolated from the grid (say after the action of a circuit breaker), then the phase of the household system can slowly drift away from that of the main grid. This can then lead to problems when the grid is reconnected.

(b) One of the problems is that the hunting action of the inverter as it attempts to reconnect can lead to very large transient spikes. These could lead to current surges.

A.4 The dynamical equations

(A.4.1) A system of equations describing the dynamics of the phase-locked loop controlled inverter were presented in Fleming and Hiskens [7] and take the form

$$\dot{m} = K_1(V_{set} - V_t), \tag{23}$$

$$\dot{\theta} = K_2(P_{set} - P_{gen}), \tag{24}$$

$$\dot{x} = K_3(\delta_t - \delta_p), \tag{25}$$

$$\dot{\delta}_p = \omega_p, \tag{26}$$

$$0 = V_i - mV_{DC}/V_{Base}, \tag{27}$$

$$0 = P_{set} - (P_0 R\omega_P), \tag{28}$$

$$0 = \theta - (\delta_i - \delta_p), \tag{29}$$

$$0 = x - (\omega_P - K_4\theta) \tag{30}$$

$$0 = P_{Gen} - V_{DC}I_{inv}/P_{Base}. \tag{31}$$

(A.4.2) In this system P_0, V_{Set}, V_{DC} and V_{base} are given values which are used to control the system. The gain of the system is given by m which controls the rate of response of the inverter. This systems is most easily understood in terms of the steady state, quasi-steady state and transient analysis.

A.5 Steady and quasi-steady state analysis

(A.5.1) We consider a system in a quasi-steady state which may either be connected to the grid or isolated from it (say after the action of a circuit breaker).

(A.5.2) In the steady state we have $\dot{m} = \dot{\theta} = \dot{x} = 0$. This gives

$$V_t = V_{Set}, \tag{32}$$

$$P_{Gen} = P_{Set} = P_0 - R\omega_P, \tag{33}$$

$$\delta_t = \delta_P \tag{34}$$

$$\theta = \theta_0, \quad m = m_0, \quad x = x_0. \tag{35}$$

(A.5.3) We deduce that in this case

$$\omega_P = x - K_4\theta = const.$$

Similarly,

$$P_{Gen} = P_0 - R\omega_P = const.$$

Hence the phase drifts at a constant rate so that

$$\delta_P = \delta_T = a + \omega_P t.$$

Thus there is the possibility of a steady drift away from the grid reference if a circuit breaker is open. Closing the circuit breaker may then lead to a transient spike.

A.6 Dynamics

(A.6.1) A reduction of the equations for the frequency ω leads to the following second order differential equation

$$\frac{1}{K_2}\ddot{\omega}_P + K_4R\dot{\omega}_P + K_3R\omega_P = K_3(P^0 - P_{Gen}). \tag{36}$$

Using values for the constants given in the literature we have

$$\ddot{\omega}_P + 80\dot{\omega}_P + 160\omega_P = 400(P^0 - P_{Gen}), \tag{37}$$

with the steady solution $\omega_S = 2.5(P^0 - P_{Gen})$

(A.6.2) The unforced equation has the solution

$$\omega = Ae^{-2.05t} + Be^{-77.9t},$$

so that there is a natural recovery time of 1/2s. However, the large disparity in the eigenvalues (the system is highly stiff and non-normal) gives a hint that there may be significant transient spikes before the system stabilises to ω_S. (Note the existence of transient spikes arising for the same reason in the DSTL problem also presented at the Cardiff Study Group.) Indeed, if P^0 is changed suddenly (due to the opening of a circuit breaker) then we would expect to see large spikes generated by the inverter as it hunts for the steady state solution. Such transient spikes are reported in the literature [7] as a major problem with inverter systems.

A.7 Conclusion

(A.7.1) The problem of using inverters is that if they drift, then re-synchronising with the network leads to large transient spikes, which could themselves be very destabilising. They might also lead to large current transients leading to local failure of the network due to the action of circuit breakers. It is this lack of inertia in the system which is likely to cause problems for future operation.

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