

# A Case Study in the Future Challenges in Electricity Grid Infrastructure

Marjan van den Akker (Utrecht University), Herman Blok (University of Leiden), Chris Budd (University of Bath), Rob Eggermont (Eindhoven University of Technology), Alexander Guterman (Moscow State University), Domenico Lahaye (TU Delft), Jesper Lansink Rotgerink (University of Twente), Keith W. Myerscough (CWI Amsterdam), Corien Prins (Eindhoven University of Technology), Thijs Tromper (University of Twente), Wander Wadman (CWI Amsterdam)

## Abstract

The generation by renewables and the loading by electrical vehicle charging imposes severe challenges in the redesign of today's power supply systems. Indeed, accommodating these emerging power sources and sinks requires traditional power systems to evolve from rigid centralized unidirectional architectures to intelligent decentralized entities allowing a bidirectional power flow. In the case study proposed by ENDINET, we investigate how the penetration of solar panels and of battery charging stations on large scale affects the voltage quality and loss level in a distribution network servicing a residential area in Eindhoven, NL. In our case study we take the average household load during summer and winter into account and consider both a radial and meshed topology of the network. Our study results for both topologies considered in a quantification of the levels of penetration and a strategy for electrical vehicle loading strategy that meet the voltage and loss requirements in the network.

keywords: power systems, load flow computations, distributed generation, electrical vehicle charging

## 1 Introduction

The problem brought to SWI2012 by ENDINET is the hugely important question of the future performance, stability and integrity of the power supply network. The issues facing power generation are changing rapidly. Until recently we have had a situation of a small number of large suppliers of electricity (typically power stations delivering 100MW or more of power to consumers with high demand during the day and low demand at night. In the future, and with the coming of the smart grid, this will change. In particular we will see a large number of small scale generation (and storage) of power (in the range of 1-10kW) from households, typically through solar cells or batteries, combined with a much

larger drain on the network at night due to the charging of electrical vehicles (at a rate of 3kW per vehicle). The increase in solar power usage for instance is illustrated in Figure 1. Both the generation and the new loads on the system substantially change the dynamics of the grid. Various questions then arise, in particular, (i) will the grid be able to cope with the new loads imposed on it without a significant change in the quality of the voltage, (ii) what are the optimal strategies for charging electrical vehicles, and (iii) will the grid supply remain stable under the unpredictable situation of variable power generation and load.

Some of these issues have been considered in previous study groups. For example the 2010 Study Group in Amsterdam, NL, looked into the optimal distribution of decentralized power generation under network constraints [5]. The 2011 Study Group in Cardiff, UK, addressed a stability question and studied in particular the behavior of the inverter units which couple the solar cells to the grid [1]. It attempted to match the phase of the locally generated AC with that of the grid.

In the SWI 2012 meeting we considered the quasi-static problem in which the grid is assumed to be in equilibrium at each time, with a slowly evolving load and generation profile (see e.g. [3]). In this context, the study group considered the question of voltage quality under varying load and supply, and looked into strategies for the optimal charging of electrical vehicles (EVs). The group examined the effect of different loading patterns by considering each household to have a time-varying stochastic load. It also looked into how either adding or deleting an important conducting line affects the voltage quality. To this end repeated load (or power) flow simulations of the voltage amplitude and phase at each nodes (or bus) in the network were performed. Computations were performed using the package MATPOWER [8] on a relatively small model network with less than 100 households connected. However, in practice all of the techniques considered could be easily scaled up to a much larger network. For the small network considered a reasonably complete answer can be given for all of the questions raised above, and this is the subject of this report.

This report is structured as follows. In Section 2 we describe the mathematical model for load flow computations resulting in the voltage amplitude and phase in a power network. We also describe the distribution network considered as well the requirements on the voltage, currents and losses imposed. In Section 3 we describe the stochastic distribution of household load both within the network as over a year. In Section 4 we describe how the penetration of solar panels affects the voltage quality. In Section 5 we describe a strategy that prevents the network to be overloaded by the charging of electrical vehicles. In Section 6 we present numerical results on of simulations of the distribution generation by solar panels and the loading of electrical vehicles. We end this report by giving conclusion and recommendations for future work in Section 7.

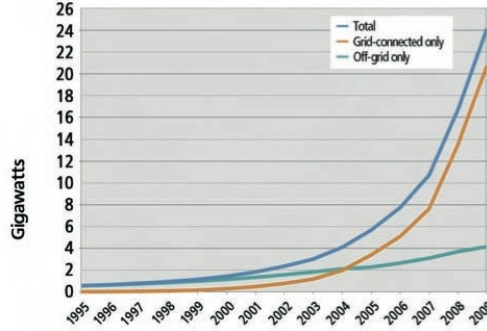


Figure 1: The increase of solar energy usage [4].

## 2 Power Flow Problem

The power flow problem is the problem to determine the voltage at each bus of a power system, given the supply at each generator and the demand at each load in the network (see e.g. [3]). The network we will consider is a low voltage network supplying a residential area consisting of a few streets. The power is fed into a network by a connection to medium voltage network through a transformer that can be regarded as an infinite source of power. The solar panels installed will be taken into account as decentralized power sources. Apart from the household loads, we will also take the loads of the charging of vehicles into account.

Let  $Y = G + jB$  denote the network admittance matrix of the power system. Then the power flow problem can be formulated as the nonlinear system of equations

$$\sum_{k=1}^N |V_i| |V_k| (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) = P_i, \quad (1)$$

$$\sum_{k=1}^N |V_i| |V_k| (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) = Q_i, \quad (2)$$

where  $|V_i|$  is the voltage magnitude,  $\delta_i$  is the voltage angle, with  $\delta_{ij} = \delta_i - \delta_j$ ,  $P_i$  is the active power, and  $Q_i$  is the reactive power at bus  $i$ . The current, voltage and power are measured in Ampère (A), Volts (V) and Watts (W), respectively. For details see again e.g. [3].

Define the power mismatch function as

$$\vec{F}(\vec{x}) = \begin{bmatrix} P_i - \sum_{k=1}^N |V_i| |V_k| (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) \\ Q_i - \sum_{k=1}^N |V_i| |V_k| (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) \end{bmatrix} \quad (3)$$

where  $\vec{x}$  is the vector of voltage angles and magnitudes. Then the power flow problem (1), (2) can be reformulated as finding a solution vector  $\vec{x}$  such that

$$\vec{F}(\vec{x}) = \vec{0}. \quad (4)$$

This is the system of non-linear equations that we solve to find the solution of the power flow problem. In our experiments we will make use of the MatPower package [8].

In our study we perform repeated load flow computations to simulate the load profile over the course of a week in either summer or winter. We seek to understand to what extent the solar panels can penetrate in the required power generation and to what affect the EVs can be loaded with out introducing malfunctions in the network.

## 2.1 Distribution Network Considered

The distribution network considered is the network with 14 busses and 14 lines shown in Figure 2. It consists of two branches. The upper branch in this figure has a meshes structure as customers in this branch are fed from more than one source. The lower branch is intentionally kept radial to make the case study more interesting. The busses are numbered consecutively from 1 to 9 and from 10 to 14 by first traversing the upper and then the lower branch in from top left to bottom right. The dotted line in the lower right part of the figure is a hypothetical that ENDINET considers building to convert the radial topology into a meshed one. The line data of the network considered is given in Table 1. Households are connected to the network by 3x25 A connections.

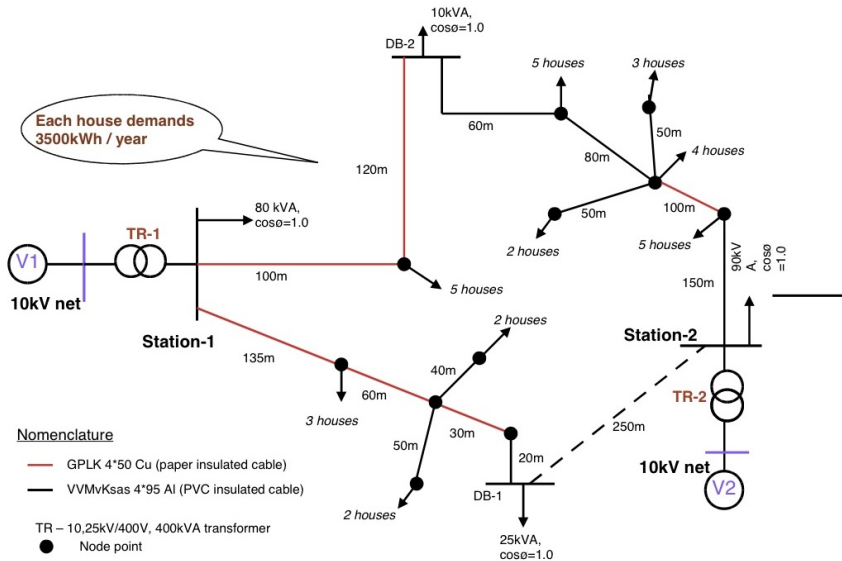


Figure 2: Diagram of the distribution network considered [2].

Cable Type	Resistance (Ohm/km)	Reactance (Ohm/km)	Maximum Current capacity (A)
Copper	0.387	0.072	170
Aluminum	0.333	0.082	195
<b>Transformer</b>	-	-	725

Table 1: Data on component in the network considered [2].

## 2.2 Network Requirements

To assess the performance of the network considered under loading, both the nodal voltages  $|V_i|$  at node  $i$  and the line currents  $|I_{ij}|$  between node  $i$  and node  $j$  need to be taken into account. Key performance indicators are:

1. the nodal voltages should be within 10% of the nominal value of 230 V, i.e.,

$$207\text{ V} < |V_i| < 253\text{ V};$$

2. the nodal voltage at any node should not vary by more than 3% between consecutive 10 minute intervals;
3. the line current  $|I_{ij}|$  on any one line should not exceed the nominal rating given in Table 1;
4. the real power losses  $|I_{ij}|^2 R_{ij}$  on any line should be sufficiently low.

In our computations, we found that the most significant reason for a loss of quality in the network was the effect of voltage variation. The limits on the current and real power loss can all be relaxed either by inserting more power sources or by avoiding radial networks with long lines. The latter indeed require higher values of current at their start node which immediately implies larger voltage drops and greater power loss. In this context radial networks are outperformed by different network topologies.

## 3 Household Load

In order to solve the power flow problem it is necessary at each load bus, to specify the real power  $P_i$  (we took the reactive power drain  $Q_i = 0$ ). We therefore have that

$$P_i = P_i^S - P_i^L - P_i^{EV}$$

where  $P_i^S$  is the power *generated* by the solar cells,  $P_i^L$  is the general household load and  $P_i^{EV}$  is the load due to the electrical vehicle charging. Each of these terms has a different form and we consider each separately.

### 3.1 Distribution of Household Load over Network

The power drain  $P_i^L$  due to consumer load is a time varying variable, that depends stochastically upon the customer. A typical household will consume around 400 W on average, with a peak load of around 1 kW. This load varies during the day (and is highest in the early evening) and we will consider the time variation in the next subsection. Similarly, the power drain varies from one household to the next, dependent, for example, on the number of people living in each house. Data for annual usage (in kWh) presented by ENDINET is shown in Figure 3. This figures indicates that over one year the total consumption  $P_i$  follows a *log-normal* distribution so that  $\log(P_i)$  has a mean of 7.904 kWh and a variance of 0.5607 kWh, i.e.,

$$\log(P_i) \sim N(7.904, 0.5607) .$$

When simulating the performance of the network the households were each assumed to follow this distribution, and the time varying household load scaled accordingly, so that the values of  $P_i^L$  in the power equation were each treated as stochastic variables with the distribution as above.

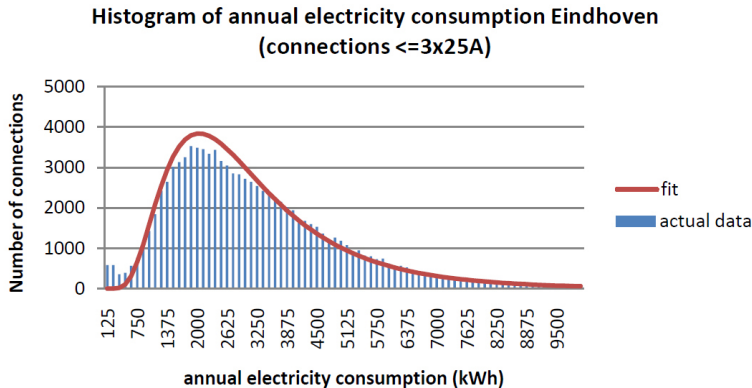


Figure 3: Annual household load distribution across the network and log-normal fit [2].

### 3.2 Time Evolution of Single Household Load

Figure 4 gives the electricity consumption of an average household as the percentage of the annual electricity consumption. The data consist of a winter and summer time series, both of one week length and a the step size of 15 minutes. Assuming a total annual electricity consumption of 3.5 kWh, we immediately transformed the data to load time series, with each data point denoting the average power consumption over 15 minutes. The winter load time series is shown in

Figure 5. An obvious observation is the diurnal periodicity of the load pattern.

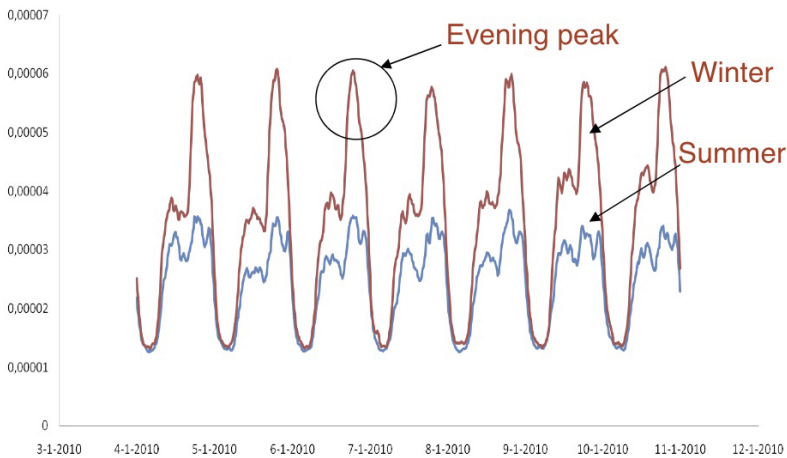


Figure 4: Summer and winter weekly profile of household load [2].

Weekdays seem to follow a comparable pattern, whereas weekend days seem to follow a slightly different pattern. In what follows, we try to capture the periodicities to obtain a *load function*. This function may be more of use than the data set itself: in our test cases e.g., we will require a load data time series on a 10 minute basis, which can be easily extracted from the load function.

**Discrete Fourier Transform** By use of the discrete Fourier transform (DFT), we find the frequency components of the winter load time series. We define an equidistant frequency grid with step size  $1/7$  per day, starting at zero frequency up till 96 per day. The double-sided amplitude spectrum is displayed in Figure 6.

To be more specific, the six most dominant frequencies (that is, those corresponding to the largest absolute values) of the single-sided amplitude spectrum are displayed in Table 2. The largest absolute amplitude corresponds to zero frequency and reflects the mean of the time series. Note that only the largest five amplitudes have an absolute value larger than 10 W. This *spikiness* suggests that the time series can reasonably well be approximated by a weighted sum of harmonics with these frequencies, which is illustrated in Figure 5.

## 4 Distributed Generation by Solar Panels

A first key question posed to the Study Group was to investigate the possible penetration of the use of solar panels. This required looking into the usage

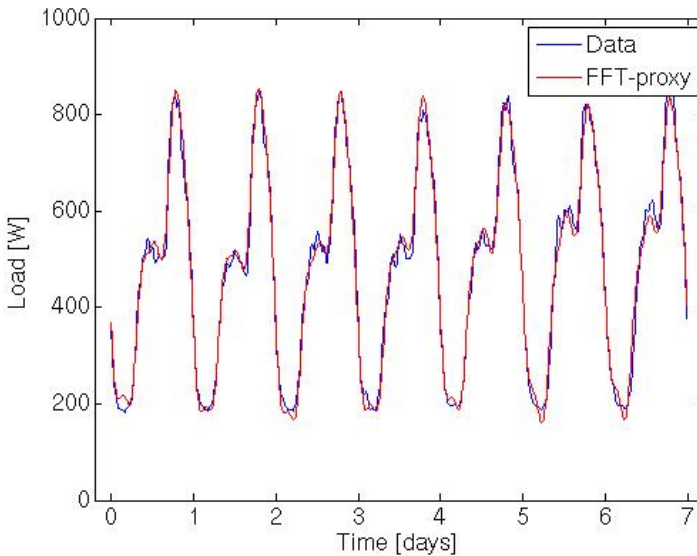


Figure 5: Time series and discrete Fourier transform approximation of load at an average Dutch household, from Monday January 4th 2010, 00:00 a.m. until Sunday January 10th 2010, 11:45 p.m.

and performances of such panels. On a good sunny day a panel can deliver a maximum of 3 kW at mid-day, but the performance of this panel depends greatly upon both the level of cloud and the the time of day and the time of the year. On an average sunny day the greatest power output is at mid-day with a rise from dawn and a fall to dusk. On a cloudy day the output will vary rapidly. This rapid variation is a potential problem as it can lead to large voltage fluctuations which can then lead to a degrading of the network as a whole. This effect will be illustrated further in this report. The variation in the illumination level of a panel, termed the *insolation* is tabulated. For example in Eindhoven, NL, it is 4.24kWh/m<sup>2</sup> per day in August, and 0.74 kWh/m<sup>2</sup> per day in the January. These figures allowed us to scale the values given by ENDINET for both summer and winter.

Accordingly in the simulations we assumed that a percentage  $p$  of the households were using solar panels and that the usage of such panels per household was a uniformly distributed random variable. Four cases were considered (using the data supplied by ENDINET) namely sunny and cloudy days in both summer and winter.



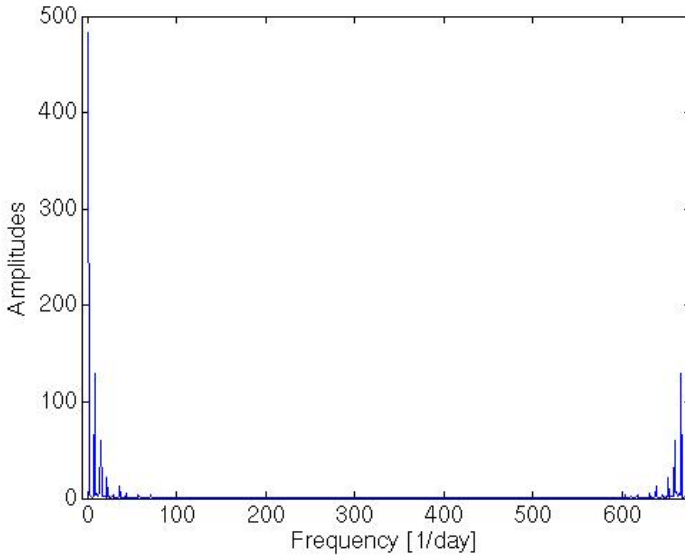


Figure 6: Double-sided amplitude spectrum of the winter load time series.

## 5 Loading by Charging of Electrical Vehicles

A second key questions posed to the Study Group was to investigate whether or not the given network can cope with the charging of EVs. The difficult part in modeling the loading by charging of EVs in the network, is the modeling of the behaviour of people in the sense of when they charge their cars and of course how much they have been driving that day. For simplicity we assume that everybody in our network gets home at 18.00 hours and immediately plugs in their car, which might be assumed as some kind of worst case scenario, because of course at 18.00 there is also a peak in household load. Another assumption is

frequency [day <sup>-1</sup> ]	amplitude  [W]	arg(amplitude) [rad]
0	482.3	0
1	129.3	1.78
2	59.8	1.73
3	20.9	-2.52
5	13.2	1.53
1 + 6/7	7.2	-1.94

Table 2: Six most dominant frequencies of the single-sided amplitude spectrum of the winter load time series.

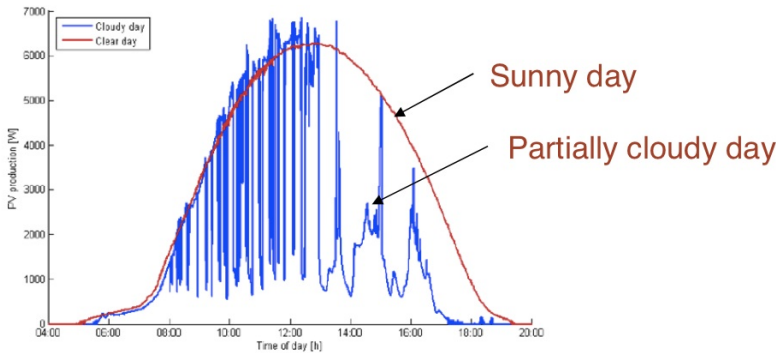


Figure 7: Typical daily power production profile from solar panels [2].

that every car actually drives the average amount of kilometers driven per person per day on a single day. This amount is about 50 kilometers and this assumption, together with the given efficiency of EVs of 5 km/kWh gives us a charging time as will be stated below.

From previous research [7, 6], we know that simply charging when people want to is not optimal at all and thus will not be a realistic case in the future. Indeed, when all cars are charged at the same time, immediately at 18.00, the network will definitely fail due to overloading. We therefore need a strategy for charging EVs. A smart grid is then a grid that is able to implement such a strategy.

Of course future work might lead to even better strategies, which because we only look at a small piece of the network might be needed regarding large networks with a lot of EVs. Our strategy is not optimal at all because it does not allow for vehicles charging at loads lower than the maximum one nor for partial charging in case of necessity.

## 5.1 Greedy Control Strategy for EV Charging

If a customer  $j$  arrives at home and wants to charge his car we are given:

- $t_j$  the time at which the customer puts the request
- $E_j^{rest}$  the energy still left in the car
- $c_j$  the time by which the customer wants the charging to be completed.
- $E_j^{charged}$  (optional) the amount of energy that the customer wants to have after charging. The default value is that the customer wants to have the car charged fully
- $D_j$  the energy requested by the customer,  $E_j^{charged} - E_j^{rest}$

We assume that the customer plugs in his car on the network at time  $t_j$ . In a *smart grid* the network can decide when the car is actually charged. The

question is now to assign to each customer  $j$  an interval that is a subset of  $[t_j; c_j]$  during which the the car is charged. This might be generalized to a collection of intervals, if we are allowed to preempt the charging of a car. In the future, it probably will be possible to charge cars at different speeds.

The problem then becomes a complex scheduling problem. At time  $t_j$  we have to decide if the car of customer  $j$  is charged or if he has to wait. If we decide to charge the car, we have to make sure that the voltage drop is at most 3 % and that the network constraints on the voltage and current are met during the charging period. Moreover, if there are waiting requests and the power consumption in the network decreases, we have to decide whether we start charging another car. Solving this problem requires an intelligent strategy.

In our case study, we restricted ourselves to a basic case. We assume that EV-charging always takes place at 3.5 kW and takes exactly 3 hours requiring therefore 10.5 kWh in total. We also assume that charging cannot be preempted. Each household owns one car, which arrives at 18.00 and wants to be charged by 07.00 the next morning. We apply the following *greedy strategy*:

1. Initialization: set time  $t = 18.00$ , all customers get state 'Request'
2. Select from the customers with state 'Request' the one at the location with largest voltage and check if starting to charge the car of this customer is feasible subject to the network constraints.
  - If Yes, start charging this car, set state of this customer to 'Charging' and update voltage and current in the network for the charging time. If there are customers left with state 'Request' go to Step 2, otherwise we are Finished.
  - If No, set  $t$  to the next point in time were consumption decreases. If the decrease is caused by completing the charging of a car, set the state of the customer to 'Complete' Go to Step 2.

An alternative strategy is obtained by selecting a random customer from the set of 'Request' customers. If charging for this customer is infeasible in the network, we randomly select another customer. We repeat this until we have found a 'feasible' customer, or found out that no request can be fulfilled. In the latter case, we have to try again at the next point in time where energy consumption has been decreased. It is not hard to see that this approximates the situation where charging requests arrive in a random order.

## 6 Numerical Results

In this section we present numerical results illustrating the impact of distributed generation by solar panels and loading by the charging of electrical vehicles.

## 6.1 Distributed Generation by Solar Panels

**Impact on Single Node Voltage** Figure 8 shows the simulated evolution in time of the voltage of the node with four neighbours in the lower branch of the network for seven consecutive days. In this simulation, the power usage of all houses in the network was chosen to be the average power usage on a typical summer day. The bottom smooth line represents the voltage at the chosen node when no solar panels are present. The voltage at this node is significantly lower than the nominal value of 230 V, but still larger than the minimum required voltage of 207 V. The top oscillatory line represents the voltage at the same node when all houses have solar panels. The solar panels increase the voltage in the node up to 7 V, which is only a good thing, as the voltage was already quite low. The graph also shows steep jumps in the voltage. This could cause violation of voltage profile constraints. Figure 8 does not significantly change if one replaces the assumption of all house consuming an average load by a load according to a log-normal distribution.

**Impact on Overall Network Performance** For every combination of household loads and solar panel placement four characteristic days are simulated: a cloudy and a sunny day in both summer and winter. The network must satisfy all the requirements mentioned in Subsection 2.2 in all four conditions to be considered a success. We separately perform the experiment on the original network and on the network with the extra cable.

The scatter plot in Figure 9 displays the results of our Monte Carlo simulations. The solar panel penetration is plotted along the abscissa and the average household power demand is plotted against the ordinate. Both quantities disregard any information about the distribution within the network.

The household power demand has little influence on the network performance. On the other hand, solar panel penetration has a great impact on the network. For penetration levels below 70% the network is robust for any distribution of solar panels and household demand. Above 90% the network fails regardless of distribution, almost exclusively due to excessive jumps in the voltage level. For intermediate values the internal distribution of solar panels and household demand has an influence. But even then the average household usage is of little influence on network performance. We studied the influence of adding a cable (the dotted line in Figure 2) to network performance by comparing the power loss in the original network to that in the network with the extra cable added. The household power usage and solar panel distribution were taken randomly in identical fashion to before, taking an average over the four typical conditions. However only the midday conditions are considered, saving considerable computational time.

The results are presented in Figure 10. It is immediately obvious the extra cable is beneficial to network performance. The power loss over the entire network is approximately halved, regardless of solar penetration.

The only influence of solar penetration on the network losses is that for intermediate values there is more variation in the distribution of solar panels throughout the neighbourhood and consequently there is some variation in the network losses. But this holds for both network configurations equally.

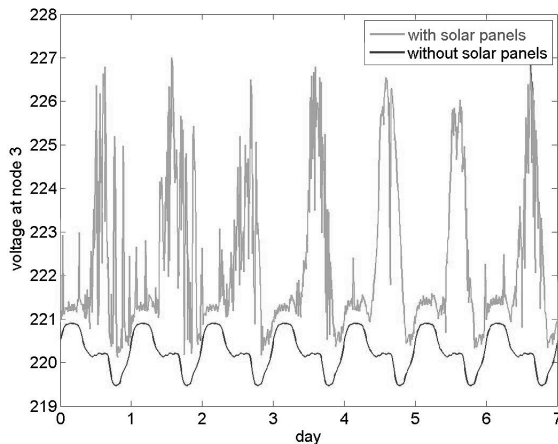


Figure 8: Voltage at the node with 4 neighbours in the lower branch of the network in Figure 2, where all houses have an average winter power usage pattern, with or without solar panels.

## 6.2 Loading by Charging of Electrical Vehicles

**Charging with no Strategy** We ran simulations for the charging of EVs between 06:00 and 22:00. We assumed average power usage on a winter day for every household. Subsequently, we randomly distributed different numbers of cars over the neighbourhood, where every household has at most one car charging at 3500W. The results are shown in Figure 11. The horizontal axis shows the number of cars in the neighbourhood. The vertical axis shows the percentage of distributions of cars that caused a network failure. For up to 10 cars, the network can still handle the load, but for larger numbers, the network may fail if these cars are wrongly distributed over the network. If all households have a car, the network will definitely fail. The same experiment was performed for simultaneous loading of cars between 24:00 and 06:00. In this simulation, no network failures occurred, for any number of cars in any distribution. Apparently the combination of high household power usage in the evening and the loading of a large number of electric cars at the same time is the cause of the problems. These results suggests that a control strategy that spreads the load

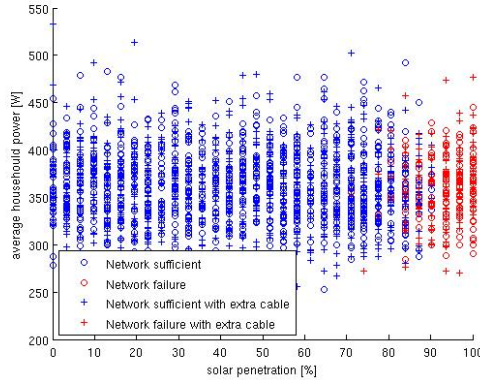


Figure 9: Scatter plot indicating network performance for combinations of solar power penetration and household power demand. Each case is run for one sunny and one cloudy day in both summer and winter. All network requirements must be satisfied in all conditions for a case to be considered a success (blue), otherwise it is considered a failure (red). The experiment was performed for the original network (o) and for the network with the extra cable (+).

due to carcharging will solve the network overload problems, while all cars can be fully charged in the morning.

**Charging with Greedy Strategy** We run simulations for the charging of EVs between 18:00 en 06:00, where we apply the greedy charging strategy described in Section 5. We assumed that everybody gets home at 18:00 and immediately plugs in their car. Each car is charged nonstop at full capacity for three hours. Each car should be fully charged before 07:00 the next morning. For simplicity, we assumed that each house has the exact same load profile. We run two simulations. One for the case that each house owns one car (Figure 12(a)), and one for the case that each house owns two cars (Figure 12(b)). Note that this second case is not realistic in the current setting, since each household has a maximum connection capacity of 25 A, thus exceeding the limit of 12 A. Charging two cars simultaneously would by itself require a capacity of 30 A. It does provide insight in the way the network handles charging that much cars using our greedy control strategy.

In Figure 12 we see that if each household owns one car, the network can handle charging almost all cars at once. Only at node three one or more cars have to await their respective turns. It takes six hours before all cars are fully charged.

If the demand is doubled, i.e. each household now owns two cars, much more planning is asked from our strategy. Now it takes nine hours before all cars are

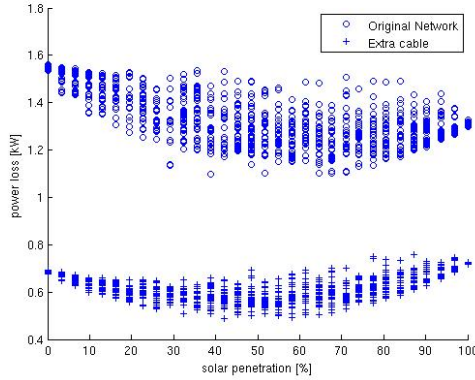


Figure 10: Network power loss in the original network compared to that in the network augmented with the extra cable.

fully charged, but still all cars are charged before 07:00.

## 7 Conclusions and Future Work

We examined the impact of the distributed generation by solar panels and of the load by the charging of electrical vehicles on the performance of a low-voltage distribution network servicing a residential area in Eindhoven, NL. Numerical results show that the penetration of solar panel usage is mainly limited by the requirement on the voltage variation between consecutive 10 minute intervals. For penetration levels above 70% the network ceases to be robust, independently of how the panels are distributed over the households. The charging of electrical vehicles requires due care to prevent that the required power amplifies the evening peak in household loading. The greedy scheduling strategy we propose allows to fully charge up to two EVs per household before 07:00. Adding the extra cable in the network allows to approximately half the power loss in the entire network.

Further investigations can be directed to the improvements of our strategies by taking into account the possibility to use different speeds of EV charging; the possibilities for partial charging and/or break-down in charging; a pricing policy for households who desire to charge an EV during the peak time; the distances driven by a car per day; other types of decentralized generators; and finally the further development of smartness of the power grid.

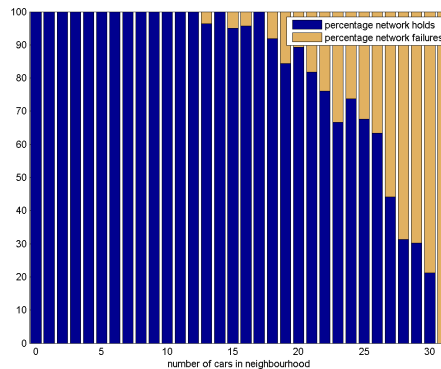
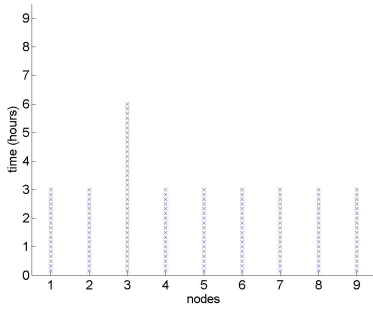


Figure 11: Percentage of distributions of cars that caused a network failure vs. number of cars attached to the network.

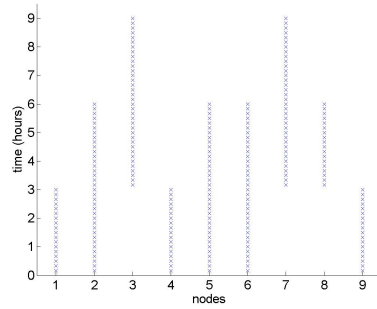
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(a) One car per household.



(b) Two cars per household.

Figure 12: EV charging profiles. The zero on the y-axis corresponds to 18:00. The x-marks at node  $i$  denote that one or more EVs are charging at that node.

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