

Cost Optimization of Ice Distribution

Poul G. Hjorth,^{*} Technical University of Denmark, Denmark

Andrew Lacey,[†] Heriot-Watt University, United Kingdom

Alessandra Micheletti,[‡] Università degli Studi di Milano, Italy

Dimitrios Tzanetis,[§] National Technical University of Athens, Greece

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Team participants

Vasileios Kostoglou, Alexander Technological Educational Institute of
Thessaloniki, Greece

Rada Novakovic, National Research Council (CNR-ICMATE), Italy
Anastasis Rossides, University of Cyprus, Cyprus

Nikolay Sterev, University of National and World Economy - Sofia, Bulgaria

Margarita Zachariou, Cyprus Institute of Neurology and Genetics, Cyprus

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Prokopis Prokopiou, CEO, Party Ice Ltd, Cyprus

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Abstract

Two questions regarding minimizing fuel costs while delivering ice along a pre-set route are tackled.

The first question is when demand exceeds the load of a single truck, so that a second truck of ice has to be taken to some point of the route for the driver/salesman to continue with that for the rest of the route: Is it better:

1) for the first truck to deliver starting from the customer nearest to the base,
or

2) for the first truck to start the delivery from the last customer (the most distant from the base)?

We show that the second strategy was better for the particular data looked at, and we have the basis of an algorithm for deciding which strategy is the better for a given delivery schedule.

^{*}email: pghj@dtu.dk

[†]Corresponding author, email: A.A.Lacey@hw.ac.uk

[‡]email: alessandra.micheletti@unimi.it

[§]email: dtzan@math.ntua.gr

The second question concerns how best to modify a regular sales route when an extra delivery has to be made. Again, the basis for an algorithm to decide how to minimize fuel costs is derived.

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1 Introduction

Party Ice is the oldest ice manufacturer in Cyprus, headquartered in the industrial Nissou area of Nicosia. The distribution of Party Ice products is carried out with a proprietary modern refrigerated truck fleet, perfectly suited for the distribution of frozen foods. The company has recently applied route control systems to achieve fuel savings and to do the best service at the best prices. By participating in the Study Group with Industry, the company aimed to make use of their available data to further optimise their distribution system and make it even more cost efficient.

Two logistics problems of delivering ice cubes in an area of Cyprus were posed.

Deliveries are generally made to a fixed set of costumers on a fixed route. When the demand for ice increases, delivery takes place using two trucks. The problem then arises as to how to optimize fuel consumption. In other words, we are looking for the best schedule for the trucks, taking into account their load (an empty truck offers lower consumption, while a full truck's consumption is higher). Two strategies are examined: 1) the first truck starts delivering from the costumer nearest to the base (depot) up to a costumer say m_1 , where the first truck gets empty, then a second full truck continues the delivery starting from the next costumer until all customers are served; 2) now the first truck starts delivering from the farthest costumer, working back to a costumer say m_2 , where it gets empty, then a second full truck continues delivering from m_2 towards the base to end the process. The study-group team analysed both strategies and concluded that the second strategy is the optimum (offering lower cost) in most cases, according to a real data set which was provided by the company. We have ignored any effect of varying speed, congestion, road gradient etc. and also assumed that delivery time does not matter (either through affecting ice quality or because of having to meet certain costumers' requirements on when their ice is supplied).

On other occasions an extra delivery must be arranged. The second problem to be looked at was again how to minimize fuel costs, now fitting in an extra sales point into an otherwise standard delivery route. Again time constraints and so on were not considered during the Study Group.

A similar problem concerning consumption is the green maritime transportation. This involves the selection of an appropriate speed of vessels, so as to optimize a certain objective like the fuel consumption, see [1]. Another related problem is that of the cold chain logistics by the national express refrigerated transport company.

If the transportation cost and the number of customers are consistent for mileage, then the longer the service mileage from the distribution centre to the customers, the greater the total transportation cost is. Therefore, the total cost of vehicle transport is proportional to the service mileage, see [4].

Similar works concerning deliveries of goods, transportation and optimization of the cost, can be found in [2, 3, 6].

The report is organized as follows. In Section 2 the description of the challenges posed by the company is described. In the next section mathematical models are discussed. In the same section, a discrete and a continuous model are studied, and the mathematical model examined to find which of the strategies minimizes the fuel cost. The following section briefly discusses the second challenge. Finally, in the last section, we summarize the conclusions and recommendations to the company.

2 Description of the Challenges

The company identified two priority challenges:

Challenge 1: Selecting the optimal distribution schedule for distant-from-base areas when using two vehicles. Each driver is responsible for a particular route each day. On the occasion his supplies are depleted before completing his route, a new fully loaded truck is dispatched to a location near his next delivery stop. The driver then continues his route with the newly stocked truck while the empty truck returns to base (Nicosia). In the case that deliveries to distant-from-base regions (e.g. Famagusta) require a second truck, it is important to fix the schedule over the delivery route in order to optimize fuel costs. One strategy is to begin by delivering at the customer closest to the distribution centre (the base), having the benefit of progressively unloading the truck, hence potentially saving on fuel costs due to the overall lower truck weight. A second strategy is to drive to the furthest destination and begin delivering from there, coming backwards towards the distribution centre.

Challenge 2: Short-term handling of urgent orders. The choice of optimal route based on geographical location is often disrupted due to urgent (unplanned) orders. The company would like to identify how to optimally cater for these urgent requests in the short term (how to alter the route for one day). Certain rural areas are not visited every day. Hence, if a customer from such a region has an urgent request the driver has to modify his route and leave his assigned district to make the urgent delivery. In that case the driver must optimize his route by selecting at what point he should deviate from his normal schedule to make the extra delivery, before returning to the regular route at the next normal delivery point.

3 Mathematical Models and Analysis for Challenge 1

Trucks always leave the base in Nicosia fully loaded, carrying 3 tonnes of ice, and usually one truck is sufficient to supply the requested amount of ice per day. How-

ever, in summertime (July and August where there are high temperatures), the demand sometimes increases, and a second truck must then be sent to deliver extra ice. The second truck is again sent fully loaded.

The key assumption we make is that labour expenses are fixed so that the varying costs will be dominated by the fuel used by the trucks. We may then take the cost associated with a day's delivery to be equivalent to the amount of fuel used. It is then clear, for a delivery needing only one truck, and with mileage (km/litre achieved by a truck) decreasing with load, that it is best to make deliveries as a route is driven away from base, the driver then returning with any remaining ice on the truck from the farthest point of the route directly to base.

3.1 Discrete-demand model

We order the delivery sites visited on a route from 1 to n , where site 1 is the closest to the base on the route, and site n is the farthest. The base can be thought of as being both site 0 and site $n + 1$. Each site, a village, can have more than one customer to be served. The delivery cost, i.e. fuel used, for a route is then given by going from base (site 0) to the first delivery point, travelling between successive sites, and then returning directly to base (site $n + 1$).

Where this problem begins to be interesting is that typically a route will require more than a single truckload. Thus, a second truck is necessarily involved.

We examine two strategies, see Figure 1.

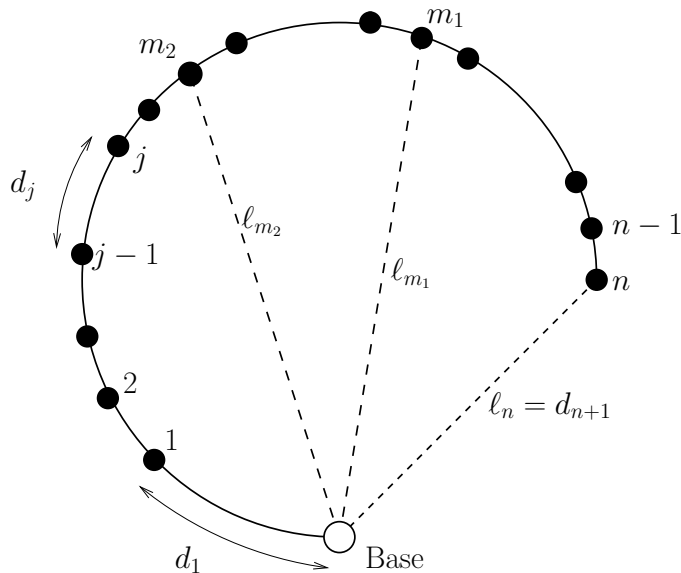


Figure 1: The representation of the two strategies.

Strategy 1: Truck 1 goes to point 1 and begins delivery. It then runs out at

point m_1 , and is there met by a full truck 2, sent out (directly) from the base. The empty truck 1 returns (directly) to base, and truck 2 completes the delivery. Truck 2, still partially loaded, returns to base.

Strategy 2: Truck 1 goes to the end point n and begins delivery in the reverse order. It will then run out at a point m_2 . It returns directly to base. A full truck 2 goes straight to point m_2 , completes the delivery, and returns to base, again partially loaded.

To write formulas for fuel used for the two strategies, we introduce some parameters and variables.

The routes are firstly characterized by distances along the segments: d_j is the distance j between point $j - 1$ and point j .

D_j is the mileage, [km/litre] over segment j . The mileage depends on how much ice is present on the truck at segment j . The mileage can vary between two limits: D_F for the full truck, and D_E for the empty truck. We always have that

$$D_F \leq D_j \leq D_E, \quad j = 1, 2, \dots, n.$$

Since D_j is dependent on strategy (whether we are at segment j as a consequence of strategy 1 or strategy 2), we add a superscript to indicate this, e.g., D_5^2 is the mileage over segment 5 with strategy 2.

Between the base and the critical, change-over, points m_1 and m_2 there are short cuts, of lengths ℓ_{m_1} and ℓ_{m_2} to/from base. These are, in general, shorter road segments, because no deliveries need be made when driving trucks directly to, or from, these points. The shortest distance between the base and the farthest delivery site, n , is denoted by ℓ_n .

In the following we assume that mileage is a linear function of the amount of ice remaining on a truck:

$$D_j = D_E + (D_F - D_E) \frac{i_j}{i_{Full}},$$

where i_j is the amount of ice being carried over segment j of the route and i_{Full} is the amount of ice carried by a full truck, namely 3 tonnes. (Cost is inversely proportional to (load + constant).) It would be possible to repeat our analysis with a more realistic variation of mileage on load, given accurate information.

With our linearity assumption, for a two-truck trip, where the load generally decreases because ice is being delivered, but at some point the truck is refilled, the mileage will be a piecewise linear function of the amount of ice delivered I , see Figure 2.

Here, I^T is the total amount of ice delivered on the route.

The values of i_j for the individual strategies can be given by subtracting off the amount of ice delivered earlier in the route:

For Strategy 1, and using tonnes,

$$i_j = \begin{cases} 3 & \text{for } j = 1 \\ 3 - \sum_{k=1}^{j-1} s_k & \text{for } j = 2, \dots, m_1 \\ 6 - \sum_{k=1}^{j-1} s_k & \text{for } j = m_1 + 1, \dots, n \end{cases},$$

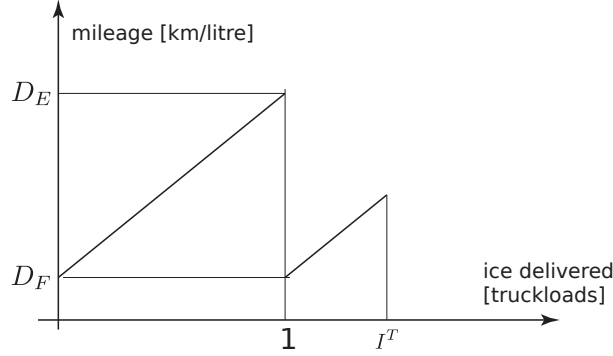


Figure 2: The graph of mileage D as a function of the ice delivered I . The unit of measure used for I is a truck load (equivalent to 3 tonnes).

where s_j is the amount of ice delivered at site j .

Likewise, for Strategy 2,

$$i_j = \begin{cases} 6 - \sum_{k=j}^n s_k & \text{for } j = 1, \dots, m_2 \\ 3 - \sum_{k=j}^n s_k & \text{for } j = m_2 + 1, \dots, n \end{cases} .$$

We can now write down the cost (fuel) functions for each of the two strategies.

Strategy 1: (delivering up to m_1) + (truck 2 going full out to m_1) + (truck 2 finishing the delivery route) + (truck 1 returning empty) + (truck 2 returning with the remaining ice):

$$C_1 = \sum_{j=1}^{m_1} \frac{d_j}{D_j^1} + \frac{\ell_{m_1}}{D_F} + \sum_{j=m_1+1}^n \frac{d_j}{D_j^1} + \frac{\ell_{m_1}}{D_E} + \frac{\ell_n}{D_{n+1}^1} .$$

(D_{n+1}^1 is the mileage for the remaining unsold ice $i(n+1) = 6 - \sum_{k=1}^n s_k$.)

Strategy 2: (driving a full truck to n) + (truck 1 delivering up to m_2) + (truck 2 going full out to m_2) + (returning empty truck 1) + (truck 2 finishing the delivery route):

$$C_2 = \frac{\ell_n}{D_F} + \sum_{j=n}^{m_2+1} \frac{d_j}{D_j^2} + \frac{\ell_{m_2}}{D_F} + \frac{\ell_{m_2}}{D_E} + \sum_{j=m_2}^1 \frac{d_j}{D_j^2} .$$

Consequently,

$$C_2 - C_1 = \ell_n \left(\frac{1}{D_F} - \frac{1}{D_{n+1}^1} \right) + (\ell_{m_2} - \ell_{m_1}) \left(\frac{1}{D_F} + \frac{1}{D_E} \right) + \sum_{j=1}^n d_j \left(\frac{1}{D_j^2} - \frac{1}{D_j^1} \right) . \quad (1)$$

We can interpret the terms as follows: the first term is the cost difference with respect to connecting the endpoint and the base; the second term is the cost difference caused by the different distances of m_2 and m_1 from the base; the third term totals the leg-by-leg cost differences.

3.2 The continuous-demand model

A continuous optimization model is based on the general model now using a supply function $S(x)$. This is the amount of ice being delivered per unit length (e.g. per km) along the route:

$$S(x) \equiv \frac{\text{demand}}{\text{length}}.$$

For convenience we now normalise length so that the total length of the route is 1. With two trucks being required, the total amount of ice delivered satisfies

$$1 < I^T = \int_0^1 S(x) dx < 2,$$

again measuring ice in truck loads.

Then the functions giving the ice supplied up to a point x for the given strategies are:

$$I_1(x) = \int_0^x S(\zeta) d\zeta \quad \text{and} \quad I_2(x) = \int_x^1 S(\zeta) d\zeta.$$

The cost function expression (1) is, in the continuous case, replaced with the expression

$$\begin{aligned} C_2 - C_1 = & \ell_n \left(\frac{1}{D_F} - \frac{1}{D(2 - I^T)} \right) + (\ell_{m_2} - \ell_{m_1}) \left(\frac{1}{D_F} + \frac{1}{D_E} \right) \\ & + \int_0^1 \left(\frac{1}{D^2(x)} - \frac{1}{D^1(x)} \right) dx. \end{aligned} \quad (2)$$

Here we have still written ℓ_{m_1} and ℓ_{m_2} as the direct distances between the change-over points and base, although sites m_1 and m_2 might now be replaced by points x_1 and x_2 respectively, likewise, ℓ_n still denotes the shortest distance between base and the furthest point on the route, and $D(2 - I^T)$ is the mileage corresponding to the load once all the ice has been delivered ($i = 2 - I^T$, using truck loads).

To analyse the continuous-function model a simple case is looked at.

3.3 Simple case: Uniform demand along a straight road

Suppose that $S(x) \equiv I^T$, a constant demand. We can in this case easily get the mileage D as a function of x in each of the two strategies. This model might be thought of as representing the supply of ice to a succession of evenly spread and closely spaced small shops or kiosks, with very similar demand, along a long road. Since supplied ice is in direct proportion to travel distance, we have:

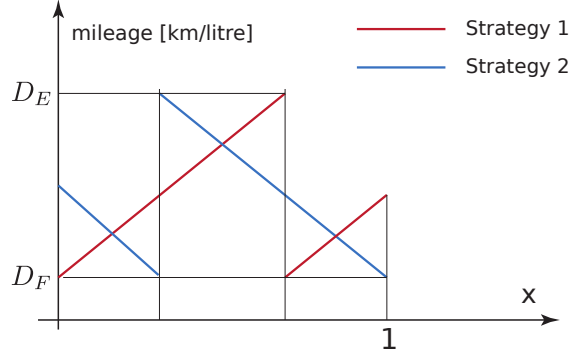


Figure 3: The position variation of mileage D for the two strategies.

$$\text{Strategy 1: Supplied ice} = I_1(x) = I^T x, \quad x_1 = m_1 = \frac{1}{I^T}.$$

$$\text{Strategy 2: Supplied ice} = I_2(x) = I^T(1 - x), \quad x_2 = m_2 = 1 - \frac{1}{I^T}.$$

Figure 3 illustrates the dependencies of D^1 and D^2 of x . Clearly the situation is symmetric, and for this reason the integral in (2) vanishes when the demand function is constant.

For extra simplicity, we also take the route to lie along a straight road leading away from the depot. Then

$$l_n = 1, \quad \ell_{m_1} = x_1 = 1/I^T \quad \text{and} \quad \ell_{m_2} = x_2 = 1 - 1/I^T.$$

The cost difference (2) then simplifies to

$$C_2 - C_1 = \left(\frac{1}{D_F} - \frac{1}{D(2 - I^T)} \right) + \left(1 - \frac{2}{I^T} \right) \left(\frac{1}{D_F} + \frac{1}{D_E} \right). \quad (3)$$

Writing $\mu = D_E/D_F > 1$, a normalised cost difference can, from (3), be written as

$$D_F(C_2 - C_1) = \left(1 - \frac{1}{\mu + (1 - \mu)(2 - I^T)} \right) + \left(1 - \frac{2}{I^T} \right) \left(1 + \frac{1}{\mu} \right). \quad (4)$$

Figure 4 shows the variation of the situation (4) as a function of total ice demand I^T when $\mu = 1.5$, which is close to the true ratio of mileages. We see that, at least for this idealised case, the costs for Strategy 2 are lower, so that this strategy is better, for I^T less than about 1.75, but for I^T greater than this Strategy 1 is the more cost effective.

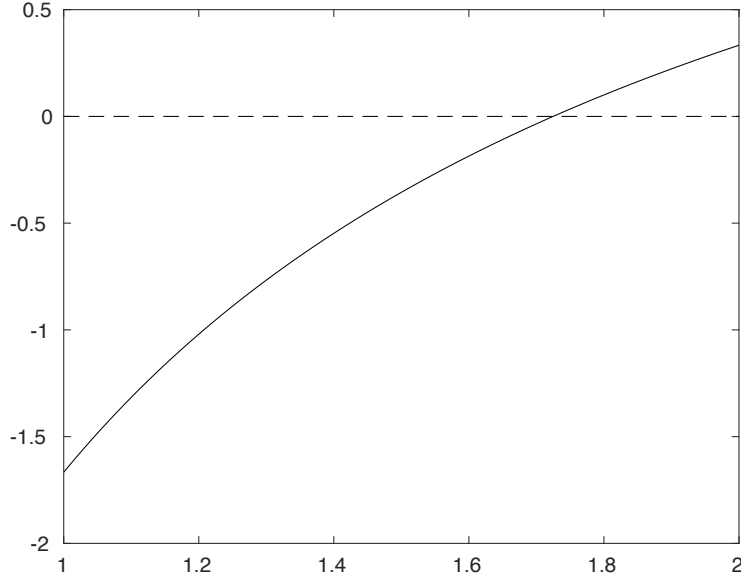


Figure 4: Normalised cost difference for mileage ratio $\mu = 1.5$.

Note that the critical value I_{cr} of I^T , such that Strategy 2 wins for $I^T < I_{cr}$ and Strategy 1 wins for $I^T > I_{cr}$, can, for this situation, be got by solving the quadratic equation

$$\mu(\mu - 1)(I - 1)I - (\mu + 1)(2 - I)[\mu - (\mu - 1)(2 - I)] = 0$$

to find the root between 1 and 2.

During the course of the study group, some other explicit demand functions $S(x)$, such as those having exponential dependence on position, $S(x) = \alpha e^{\beta x}$, were looked at. However, the calculations became rather intricate and no additional insights were gained.

3.4 Stochastic simulations of the discrete demand model

More realistic calculations were based on a genuine delivery route, with delivery at discrete sites, i.e. villages, using data supplied by Party Ice.

All trucks start from the base, located south of Nicosia, see Figure 7.

Village n.	AREA	KM FROM BASE	distance from previous stop	KILOS OF ICE	P.O.S.
1	Xylotymvou	42	42	336	2
2	Ormideia	48	6	672	4
3	Xylofagou	56	8	288	3
4	Liopetri	66	10	264	2
5	Sotera	75	9	240	1
6	Frenaros	80	5	192	2
7	Deryneia	90	10	1000	6
8	Paralimni	104	14	1800	15

Figure 5: Data provided by the company. P.O.S. (Points Of Sales) = number of customers. Party Ice also provided the amount of ice bought by individual customers but this is not displayed here.

The given example is for a Famagusta route that is between 220 and 278 km long based on the served customers. A fully deployed truck has 3 tonnes of ice. The data refers to the route from the base to Famagusta on a Tuesday, during the high season, in which the delivery of a second truck was needed. This day was considered quite representative of the usual situation. The data is shown in Figure 5. The extreme mileages are 3.9 km/litre (for a fully loaded truck, D_F) and 5.5 km/litre (for an empty truck, D_E).

Note that the demand in the last two sites is approximately 93% of a truck load, where there is a high number of points of sales (P.O.S.). This indicates that ice demand varies considerably over the route, in contrast to the uniform model of Subsection 3.3.

We decided to test, with stochastic simulations based on real data, how sensitive the cost difference (1) is to random variations in the sales, and thus how much uncertainty there is in the choice of the best strategy.

By computing the histogram of the frequencies of the customers' requests, as shown in Figure 6, we observe that the amount of ice which is most frequently sold to each customer is 96 kg. (The reason of such frequency is that most of refrigerators which Party Ice supplies to its customers contain a maximum of 96 kg of ice. A small amount of customers instead have been provided with a bigger refrigerator. These customers are the ones who usually buy the higher amounts of ice shown in the histogram.)

We then simulated the discrete model for both strategies, using the real number of sites (i.e. villages) visited by the truck during the route to Famagusta, the real distances between the sites, the number of P.O.S. reported for each site in the data, but we used two methods to introduce randomness in the demands:

- a) we generated random demands according to the sample distribution of the given data

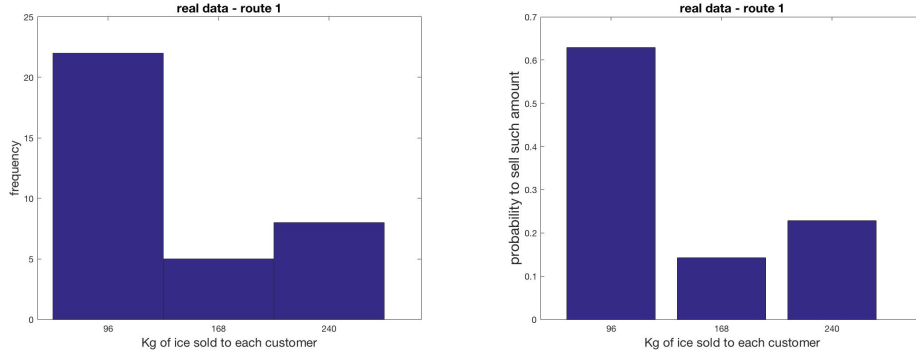


Figure 6: Distribution of ice requests from the customers. Left: absolute frequencies; right: proportions (and thus estimate of probabilities) of the requests.

- b) we generated random demands distributed uniformly around the observed data, with a fixed maximum deviation of a fraction p .

All the simulation have been performed using Matlab R2016a.

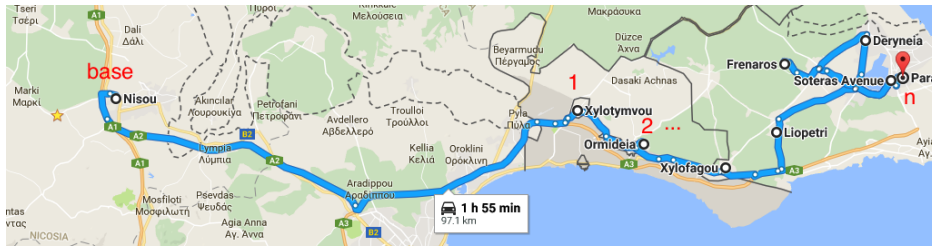


Figure 7: Working Example: Route 1 - From base to Famagusta.

3.4.1 Simulation using the sample distribution: method (a)

We generated for each P.O.S. a random number distributed according to the discrete distribution of the real data, shown in Figure 6 on the right. A random number following a given discrete distribution can be generated by the inverse transform method (see e.g. [5, Chapter 4]). Such numbers represent the ice demand of each P.O.S. in one simulation.

We performed 100 simulations of the discrete model with such random ice demands, obtaining thus 100 estimates of the costs difference (1). The result is that about 60% of times Strategy 2 is better than Strategy 1, as is shown by Figure 8.

For each simulation we also computed the total amount of ice sold when each of the two strategies wins. The distributions of the amount of sales are shown in Figure 9.

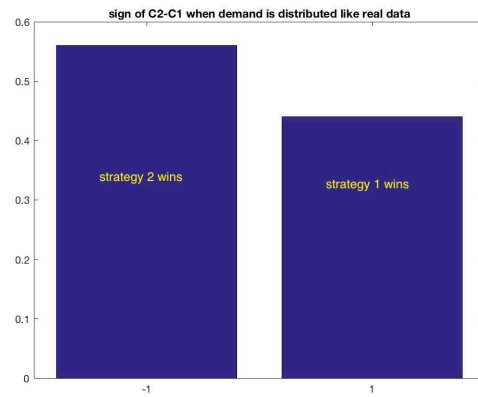


Figure 8: Distribution of the difference $C_2 - C_1$ using Strategy (a) to simulate the ice demand.

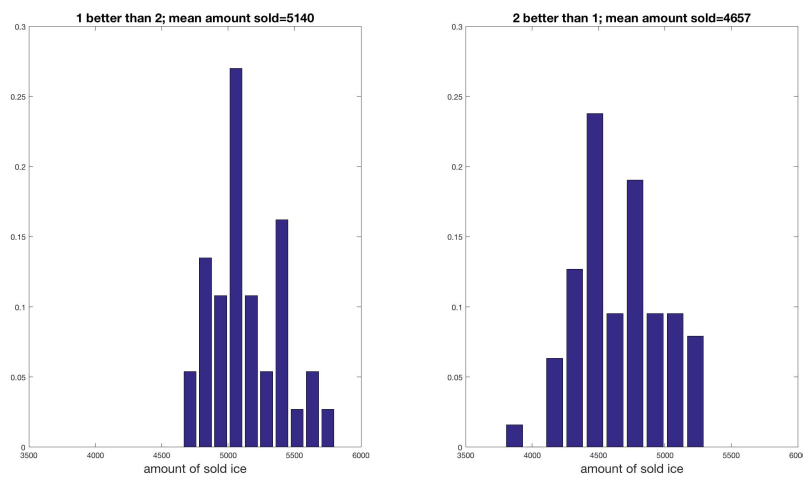


Figure 9: Distributions and means of ice sales with the two strategies. The amount of sold ice is measured in kilograms.

By observing the distributions, and taking into account that one full truck contains 3000 kg, we have that the mean ice sold when Strategy 1 wins is about 1.7 full trucks (f.t.), while the mean ice sold when Strategy 2 wins is about 1.5 f.t. Thus, roughly speaking, we could say that

- Strategy 1 is more likely to be better when sold ice > 1.7 f.t.;
- Strategy 2 is more likely to be better when sold ice < 1.5 f.t.;
- when $1.5 \text{ f.t.} \leq \text{sold ice} \leq 1.7 \text{ f.t.}$ the strategies are about equivalent.

This result is in accordance with the existence of a specific threshold of amount of sold ice over which the preference on the two strategies is switched, as we found in the study of the simplified continuous model. In the more realistic case that we simulated here, an additional “uncertainty region” is added, because of the introduced randomness.

3.4.2 Simulation using a maximum deviation: method (b)

Since the variation in ice demand is strictly connected to the size of refrigerators of each customer, and is thus not varying much when the temperature is high, we also used a second strategy to simulate the randomness, which allows one to control the maximum deviation from the typical demand values that the company observes: we took as reference amounts the data shown in Figure 5, and we added to each reference amount s_j , sold at the j -th P.O.S., a random number uniformly distributed in the interval $(-p \cdot s_j, +p \cdot s_j)$, where $p \in (0, 1)$ is a proportion of allowed maximum variation.

We then performed 100 simulations of the discrete model by fixing $p = 0.1$ (i.e. allowing a maximum random deviation of 10% from the real data), and we computed the cost difference (1) on each simulation, obtaining that about 95% of times Strategy 2 is better than Strategy 1, as shown in Figure 10.

We observe that actually for the given real data Strategy 2 is better than Strategy 1, and from our simulations we deduce that a small deviation from the given data does not influence the results much. Furthermore, considering that presently the company is using Strategy 1 for ice delivering, by computing the ratio $(C_2 - C_1)/C_1$ in each simulation, we observe that the mean cost reduction that Party Ice would obtain by choosing Strategy 2 ranges between 1.5% – 2% of the present fuel costs when the demand variation does not exceed 10% of the given data.

4 Second Challenge: An Addition Delivery Point

We now consider the question of how best to fit in an extra delivery into an otherwise normal route (with just one truck being used). The new demand might arise before the driver has started, or he might only be notified that he must fit this sale into his schedule while he is proceeding around his regular route. The extra site can be on the planned route, or outside the planned route, see either Figure 11 or Figure 12. The new delivery can be fitted into a revised schedule by the driver proceeding as

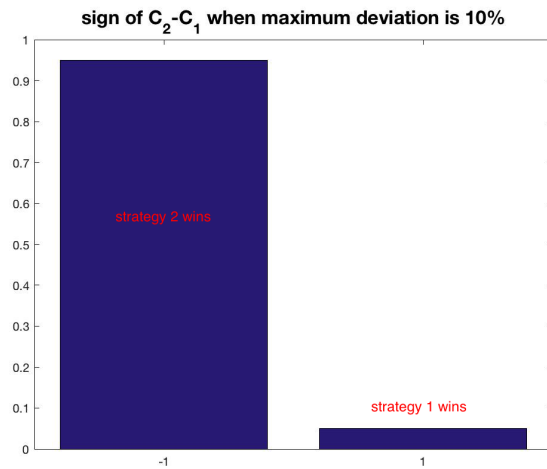


Figure 10: Sign of costs difference 1 when $p = 0.1$ over 100 simulations with simulation method (b).

normal to a site m , proceeding to the new point, say Z , returning to the regular route at site $m + 1$, and then continuing in standard fashion. The question is then how best to choose m so as to minimize costs, i.e., fuel used.

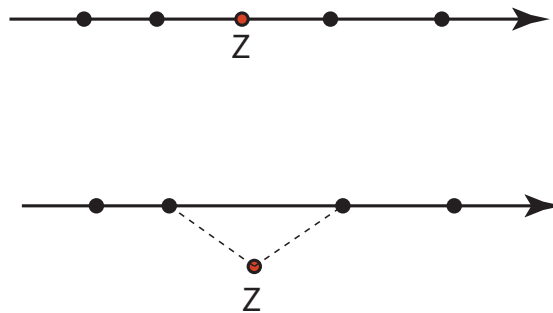


Figure 11: An additional delivery point Z .

The strategy taken was as follows: If Z belongs to the route beyond where the truck presently is, just continue along the route. If Z belongs to the route but back from the present position, or is off the route, evaluate additional the cost (using formulas similar to those for Challenge 1) got by making the diversion between sites m and $m + 1$ and then see what m gives least cost.

As in Section 3, the distance between sites $j - 1$ and j on the route is denoted by d_j . (For extra ease of notation, this now also includes d_{n+1} = distance from final (regular) site n back to base, i.e. site $n + 1$.)

The mileage $D(i_j)$ is that achieved over segment j before the detour (and on the first section of the detour). Here $i_1 = 3$ and $i_j = 3 - \sum_{k=1}^{j-1} s_j$ for $j = 1, \dots, m$, now taking $i_{m+1} = 3 - \sum_{k=1}^m s_j$ to mean the amount of ice left on the truck over the first section of the detour, which is of length d_{Z1} (see Figure 12).

The mileage $D(i'_j)$ is that achieved over segment j after the detour (and on the second section of the detour). Here $i'_j = 3 - s_Z - \sum_{k=1}^{j-1} s_j$ for $j = m + 2, \dots, n + 1$, now taking $i'_{m+1} = 3 - s_Z - \sum_{k=1}^m s_j$ to mean the amount of ice left on the truck over the second section of the detour, which is of length d_{Z2} (see Figure 12); s_Z is the amount of ice (in tonnes) delivered to the extra site.

Without the extra delivery, when the deliveries go by plan, the fuel costs are equal to

$$C = \sum_{j=1}^{n+1} \frac{d_j}{D(i_j)}.$$

With the detour between sites m and $m + 1$ to make the extra delivery, these costs, as for Challenge 1 in Section 3, become

$$C_{Zm} = \sum_{j=1}^m \frac{d_j}{D(i_j)} + \frac{d_{Z1}}{D(i_{m+1})} + \frac{d_{Z2}}{D(i'_{m+1})} + \sum_{j=m+2}^{n+1} \frac{d_j}{D(i'_j)}.$$

We then simply choose m from possibilities r, \dots, n so as to minimize the extra cost difference $C_{Zm} - C$, where site r gives the driver's present position. (If the extra delivery is known about before the driver starts then $r = 0$.)

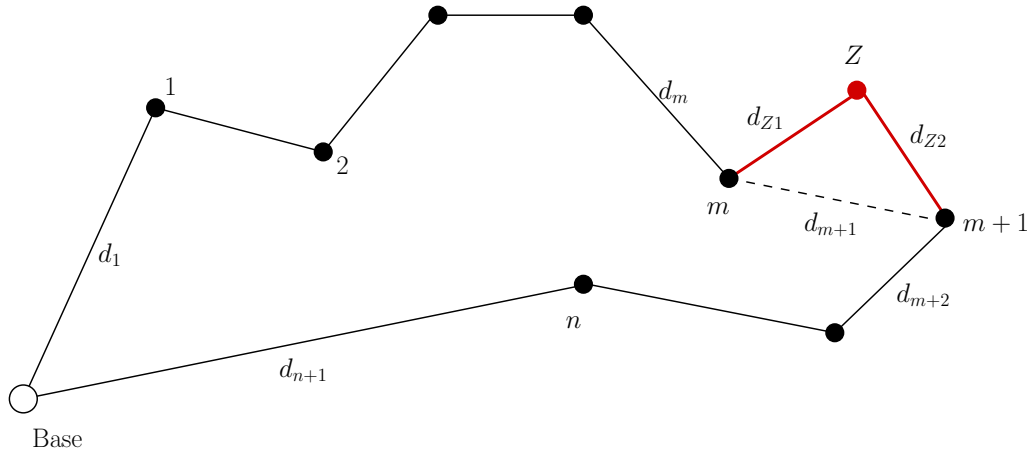


Figure 12: An additional delivery point Z between sites m and $m + 1$.

The strategy might be modified if there are time constraints on deliveries, to the regular sites or extra customer, but we have not considered these.

5 Conclusions and Recommendations to the Company

In answer to the company's first challenge, it is possible to estimate the relative costs of carrying out the "forward" and "reverse" strategies, Strategies 1 and 2, according to the modelling of Section 3. For the particular real situation considered, Strategy 2 was better. Likewise, similar modelling in Section 4 indicated how it would be possible to best fit in an extra delivery, Party Ice's Challenge 2.

It should be possible to encode the algorithms arising from the modelling so that the company can make day-to-day decisions, given appropriate information on demand etc.

Ideally, better representation of how fuel consumption depends on load, road conditions and so on should be included in the model and any algorithm. Constraints, coming from customers' requirements on when their deliveries are made might also have to be allowed for.

It is also recommended that a statistical analysis is carried out on the company's historical data of variation of demand. This will allow realistic variations to be included in codes, giving more reliable forecasts of the best strategy.

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