Selection Effects in Forensic Science

Geert Jan Franx[*] *Yves van Gennip*[†] *Peter Hochs*[‡]
Misja Nuyens[*] *Luigi Palla*[*] *Corrie Quant*[*]
Pieter Trapman[*]

Abstract

In this report we consider the following question: does a forensic expert need to know exactly how the evidential material was selected? We set up a few simple models of situations in which the way evidence is selected may influence its value in court. Although reality is far from a probabilistic model, and one should be very careful when applying theoretical results to real life situations, we believe that the results in our models indicate how the selection of evidence affects its value. We conclude that selection effects in forensic science can be quite important, and that from a statistical point of view, improvements can be made to court room practice.

1 Introduction and problem statement

At a crime scene, a red fibre is found on the victim of a murder. After the police have found a suspect, a search of his wardrobe reveals a red jumper. The jumper and fibre are brought to the forensic lab, which has to check whether the fibre matches with the jumper and if so, how *strong* this evidence is. Obviously, a very rare jumper should be considered as stronger evidence than one bought at a large company like H&M. But does the strength of the evidence depend on how and in what circumstances the jumper was found? For example, is the evidence stronger if the suspect had no other jumpers? Or does it make no difference?

This is an example of the following question posed to the Study Group by the Netherlands Forensic Institute (NFI): does a forensic expert need to know how exactly the evidential material was/is selected? This question is also relevant to the use of video identification and DNA-databases. Suppose a video recording of a crime is shown on television, and a number of suspects are brought to the attention of the police by people who watched the broadcast. Can the same video material that was used to select the suspects also be used as evidence in court? And if so, does the strength of this video evidence, for example, depend on the number of people that know the suspect? For the second example, suppose that somebody becomes a suspect because his or her DNA is in a DNA-database, and matches a DNA-sample from a crime scene. Clearly such databases do not include the DNA of the entire population. But does this

[*]Vrije Universiteit Amsterdam
[†]Technische Universiteit Eindhoven
[‡]Radboud Universiteit Nijmegen

make a difference? And should, after the selection of a suspect via his or her DNA, this DNA match be discarded as evidence?

At the moment, the situation is usually as follows. When the evidence is presented, the fact that a suspect has been selected using, for example, video material, is considered not to influence the value of the video material as evidence. Neither is the number of clothes in someone's wardrobe taken into account when fibres found on a crime scene match with a suspect's clothes and are used as evidence. This is somewhat alarming, since the judge cannot be expected to have knowledge of statistics, and possible statistical corrections should be made before the evidence is presented to him. On the other hand, the NFI expert handling the case is also not a statistician. (S)he has the possibility to call in the help of statisticians, but does so only if (s)he feels the need to do that. So the question posed to the Study Group can be seen as a request for help in dealing with situations that seem straightforward, but in fact may need statistical corrections.

In this report we answer this question in the following way. We set up a few simple models of situations in which the way evidence is selected may influence its value in court. We then give expressions for the probability that the evidence found indeed originates from the suspect, given that lab tests, or a broadcast of video material, links the evidence to the suspect. From that we conclude if and how the selection of the evidence influences this probability. Obviously, reality is far from any probabilistic model, and one should be extremely careful when applying theoretical results to real life situations. This is especially dangerous when statistics is used to prove that a crime has been committed, see for example Van Lambalgen and Meester [4]. Therefore, in the models discussed in this report this situation is excluded by assuming that it is sure that a crime has been committed. Keeping these warnings in mind, we nevertheless believe that our simple models can teach us something about how the evidence selection procedure affects the value of the evidence.

The report will adhere to the following structure. In Section 2 we explain the notion of conditional probabilities and Bayes' rule, which are used later on. A judge might want to have an indication of the probability that a suspect is guilty given certain evidence. Bayes' rule allows us to express this probability in terms of the probability that the evidence is found when the suspect is guilty, the probability that the evidence is found when the suspect is innocent, and a so-called *prior*. In Section 3 we consider a simple model of the situation where the police find a fibre on a crime scene that matches a jumper from a suspect's wardrobe. We argue that the number of jumpers the suspect owns influences the evidential value of the fibre, and describe in what way it does. In Section 4 we look at video evidence. Under simplifying assumptions, we compute the probability that a suspect who is recognised on video, is actually the person on the video material. Finally, in Section 5 we consider a different aspect of the selection of evidence, namely the influence of non-matches between the suspect and the available evidence. We argue that these non-matches may count as negative evidence, and should therefore be taken into account as well. We finish the report with some conclusions.

2 Bayes' rule and likelihood ratios

During a trial, the scientific analysis of the evidence found on the crime scene is reported to the court by the forensic expert. In order to provide the judge with the means to evaluate such evidence, the law allows/requires the forensic scientist to summarise his expertise by means of a *likelihood ratio* (see [2]). Define

$$E := \text{“Evidence at crime scene is matched with the suspect”;}$$
$$H := \text{“The suspect himself left the evidence”.}$$

The likelihood under the hypothesis that the suspect is guilty is given by the conditional probability $P(E \mid H)$; the likelihood under the (null) hypothesis that the suspect is innocent is given by $P(E \mid H^c)$. matching the suspect at the crime scene, then $P(E \mid H) = 1$. Hence, the *likelihood ratio* is defined as

$$LR = \frac{P(E \mid H)}{P(E \mid H^c)}.$$

Since the numerator is likely to be large, the crucial task of the forensic scientist boils down to assigning a value to the denominator of the above formula. This means evaluating the probability of a random match. In this evaluation procedure lies the difficulty in reporting evidence, since the likelihood under the null hypothesis depends on the assumptions made on the reference population, which might be hard to define.

A formalisation of the inferential process performed by the judge/juror is well expressed by Bayes' theorem ([1]), one of the basic formulae in probability theory. This theorem was first proposed by rev. Thomas Bayes in the 18th century and constitutes the basis for the 20th century Bayesian school of statistical inference ([5]), as opposed to the classical or frequentist school. Bayes' theorem (also named Bayes' rule for its simplicity) expresses the *posterior* probability in terms of likelihoods and *prior* probabilities:

$$\begin{aligned} P(H \mid E) &= \frac{P(H \cap E)}{P(E)} = \frac{P(E \mid H)P(H)}{P(E)} \\ &= \frac{P(E \mid H)P(H)}{P(E \mid H)P(H) + P(E \mid H^c)P(H^c)}. \end{aligned} \tag{1}$$

Here the denominator in the last term follows from the rule of total probability. The appeal of this formulation is due to a foundational inference argument, namely that the actual matter of interest in the inductive process is the posterior probability. This is the probability of the hypothesis (H) given the evidence (E), and not the likelihood or inverse probability, which is the probability of the evidence (E) given the hypothesis (H). In particular, it is the conditional event $H \mid E$ that the judge will, more or less consciously, probabilistically evaluate to give the sentence, while the scientist presents his probability estimate on $E \mid H$. Dividing the numerator and denominator of (1) by $P(E \mid H)P(H)$, we obtain the following formula:

$$P(H \mid E) = \left(1 + \frac{P(H^c)}{P(H)} \frac{P(E \mid H^c)}{P(E \mid H)}\right)^{-1}. \tag{2}$$

We can see how the posterior probability depends on the *prior odds*, i.e., the ratio $P(H^c)/P(H)$ between the a priori probabilities of the hypotheses and the inverse of the likelihood ratio as defined above. The prior odds give a formal way to incorporate non-statistical evidence into the model. Observe that although it is tempting to think so, a large likelihood does not automatically imply a large probability that the suspect is guilty.

Bayesian inference is a generalisation of the exclusively likelihood based classical inference. The relevance of a Bayesian modelling approach is also more appropriate when dealing with unique cases and often limited pieces of evidence as in the forensic setting, where doing justice cannot be achieved via the long run philosophy underlying the classical approach ([3]). For this reason, although the forensic scientist cannot or is not allowed to specify priors on the suspect's guilt, it is useful for him to summarise the judicial inductive process before presenting results in court, as will be seen in the following section.

3 The jumper model

In this section we consider a simple example that shows that for computing or estimating the probability that a suspect is guilty, it is necessary that the information about the evidence and the way it is selected should be as complete as possible. The example we give is not too realistic, but it teaches us a lot about more realistic models, and partly answers the NFI question.

3.1 Introduction

The case is as follows. Suppose an event has taken place at which someone, (the *donor*) has left a fibre of his jumper on some other person (the *victim*). We do not know anything about this donor. In order to find the donor, we investigate the jumpers in the wardrobe of an arbitrary person (for notational convenience, this person is called the *suspect*). We find a jumper made of a fibre of exactly the same type as was found on the victim. We want to compute the probability that our suspect is in fact the real donor of the fibre on the victim, given the evidence found, i.e., given that the fibres of the suspect and those on the victim match. The question we ask ourselves is how much we need to know about the evidence found: is it sufficient to know that one of the jumpers of the suspect matches the fibre on the victim? Or do we for example also need to know how many of his jumpers consist of fibres of other types?

In Section 3.2 we specify our assumptions. These allow us to compute the probability that our suspect is the donor in Section 3.3. In Section 3.4 we generalise this model to the case that more than one fibre is found on the victim. Finally, Section 3.5 deals with our conclusions and possible extensions of the model.

3.2 Assumptions and notation

We make the following assumptions to model the situation described above.

1. Only one fibre is found on the victim; this fibre is of type Y and does not belong to the victim. (We assume all fibres in the world can be categorised into

a number of different types that the forensic expert can tell apart.)

2. The fibre found on the victim was transferred during a meeting between the victim and the donor, and originates from the donor's jumper. We call the moment of this meeting the *transfer moment*.

3. Since the transfer moment nobody has thrown away or hidden any jumpers.

4. Every jumper consists of only one type of fibre.

5. The relative frequency of the total population wearing jumpers of a specified fibre type at the transfer moment is known. In particular, the relative frequency of the population wearing a jumper of fibre type Y is g_Y. This can be interpreted as the probability that some random person was wearing a jumper of fibre type Y at the transfer moment.

6. The probability that the suspect was wearing a jumper of fibre type Y at the transfer moment is known and denoted by f_Y.

7. The collection of jumpers of any person is independent of him being the donor or not.

Furthermore, we write E_1 for the event that the fibre found on the victim is of type Y and E_2 for the event that we found a fibre of type Y in the collection of jumpers of the suspect. The event that the suspect is the donor of the fibre on the victim is denoted by D.

3.3 Computations

We are ready to compute the probability that our suspect is the donor of the fibre on the victim given the evidence found. Using (2), we compute

$$\begin{aligned} P(D \mid E_1 \cap E_2) &= \left(1 + \frac{P(D^c)}{P(D)} \frac{P(E_1 \cap E_2 \mid D^c)}{P(E_1 \cap E_2 \mid D)}\right)^{-1} \\ &= \left(1 + \frac{P(D^c)}{P(D)} \frac{P(E_2 \mid D^c)P(E_1 \mid D^c \cap E_2)}{P(E_2 \mid D)P(E_1 \mid D \cap E_2)}\right)^{-1} \\ &= \left(1 + \frac{P(D^c)}{P(D)} \frac{P(E_2)g_Y}{P(E_2)f_Y}\right)^{-1} = \left(1 + \frac{P(D^c)}{P(D)} \frac{g_Y}{f_Y}\right)^{-1} . \end{aligned}$$

For third equality we used assumptions 5 up to 7. Observe that if g_Y is smaller (the fibre is rare), the probability of the suspect being the donor is larger. Also, if f_Y is smaller (the suspect does not wear the jumper of fibre type Y often), the probability of the suspect being the donor is larger. In the special case that the suspect owns k jumpers and wears those with equal frequency, we have $f_Y = 1/k$. Hence, the more jumpers the suspect has, the smaller is the probability that he is the donor (in this report we use male pronouns for the suspect and the donor). This seems quite reasonable: a person owning thousand jumpers is very likely to match with the fibre on the victim, but the strength of this match is, of course, very low.

3.4 The jumper model with more than one fibre

In this section we consider the situation that a crime is committed at which the offender *possibly* donated a fibre of his jumper to the victim. We find a number of types of fibres at the crime scene, which surely originate from some jumpers. Based on the place at the crime-scene where a particular fibre is found, we may assign some probability to the event that that particular fibre was donated at the moment of the crime. As in the previous example, we investigate the wardrobe of a suspect to see if there are jumpers with fibres that match with fibres found on the crime scene.

We are interested in the questions "does the probability that the suspect is the offender, given that there is a match, depend on the number of fibres found at the crime scene?" and "does this probability depend on the number of matches found?"

Our assumptions are the same as in the previous section with the following exceptions.

1'. There are n types of fibres found at the victim, the ith type of fibre is called Y_i for $i \in \{1, \ldots, n\}$.

2'. The *a priori* probability that the ith type of fibre is connected to the crime is h_i. These h_is are independent of the identity of the donor, if we do not take his collection of jumpers into account.

8. At most one person donated a fibre at the crime, so there is only one offender; if there is a donor, he donated at most one fibre.

From these assumptions we see that $\sum_{i=1}^{n} h_i \leq 1$. Note that it is possible that none of the fibres is left by the offender, the probability of that event is $1 - \sum_{i=1}^{n} h_i$.

Write $E_1^{(i)}$ for the event that the i-th type of fibre on the crime scene is of type Y_i and $E_2^{(i)}$ for the event that a fibre of type Y_i is in the collection of jumpers of the suspect. The event that the i-th type of fibre was donated at the moment of the crime is $C^{(i)}$ and the event that the suspect is the donor of the fibre donated at the moment of the crime is D^*. Write V for the collection of fibre types at the crime scene and W for the collection of fibre types from jumpers of the suspect. Denote the intersection of V and W by A, so A is the set of types of fibres that are both in the collection of the suspect and at the crime scene (the matches).

We are interested in $P(D^* \mid \bigcap_{j \in V} E_1^{(j)}, \bigcap_{j \in W} E_2^{(j)})$, the probability that the suspect donated the fibre at the moment of the crime, given the evidence. Because the events $C^{(i)}$ are disjoint, and $D^* = \cup_i C^{(i)}$, we can write

$$\begin{aligned}
P(D^* \mid \bigcap_{j \in V} E_1^{(j)}, \bigcap_{j \in W} E_2^{(j)}) &= P(\bigcup_{i \in V} \{D^* \cap C^{(i)}\} \mid \bigcap_{j \in V} E_1^{(j)}, \bigcap_{j \in W} E_2^{(j)}) \\
&= \sum_{i \in V} P(D^* \cap C^{(i)} \mid \bigcap_{j \in V} E_1^{(j)}, \bigcap_{j \in W} E_2^{(j)}) \\
&= \sum_{i \in A} P(D^* \cap C^{(i)} \mid E_1^{(i)} \cap E_2^{(i)}).
\end{aligned}$$

Here we used that the event $\{D^* \cap C^{(i)}\}$ is independent of $E_1^{(j)}$ and $E_2^{(j)}$ for $i \neq j$ and that the summand is 0 if the fibre at the crime scene is not in the collection of jumpers of the suspect. We can use the results of the previous section to conclude that:

$$
\begin{aligned}
P(D^* \mid \bigcap_{j\in V} E_1^{(j)}, \bigcap_{j\in W} E_2^{(j)}) &= \sum_{i\in A} P(C^{(i)} \mid E_1^{(i)} \cap E_2^{(i)}) P(D^* \mid C^{(i)} \cap E_1^{(i)} \cap E_2^{(i)}) \\
&= \sum_{i\in A} h_i \left(1 + \frac{P(D^{*c} \mid C^{(i)})}{P(D^* \mid C^{(i)})} \frac{g_{Y_i}}{f_{Y_i}}\right)^{-1} \\
&= \sum_{i\in A} h_i \left(1 + \frac{P(D^{*c})}{P(D^*)} \frac{g_{Y_i}}{f_{Y_i}}\right)^{-1},
\end{aligned}
$$

where the second equation holds since under the condition that a certain fibre was donated at the crime, everything is the same as in the situation where only one fibre is found. The last equation holds since the probability of being the donor does not depend on the fibre found if nothing is said about the collection of jumpers of the suspect. Note that the probability that the suspect is the offender may be larger than the probability computed, because he may have committed the crime, while he did not donate any fibre.

If it is certain that one of the fibres found was donated at the moment of the crime and if all fibres have the same probability to have been donated at the moment of the crime, $1/n$, we get

$$
P(D^* \mid \bigcap_{i\in V} E_1^{(i)}, \bigcap_{i\in W} E_2^{(i)}) \quad = \quad \sum_{i\in A} \frac{1}{n} \left(1 + \frac{P(D^{*c})}{P(D^*)} \frac{g_{Y_i}}{f_{Y_i}}\right)^{-1}.
$$

So the probability that the suspect is the offender decreases when the fraction of fibres on the victim that match with jumpers in the collection of the suspect decreases.

3.5 Conclusions and possible extensions of the model

In our (very basic) model we have shown that many things should be reported in order to interpret the evidence well. Not only that a jumper in the collection of the suspect and a fibre found at the crime scene match, but also how often the suspect wears that particular jumper and how many fibres are found at the crime scene. We have shown that the number of jumpers in the wardrobe and the number of fibres at the crime scene influence the probability that the suspect is the offender.

In this section we have analysed a very basic example dealing with evidence in our model. We were forced to make many assumptions in order to get some results. In the future, efforts can be made to relax some of the assumptions. For example, one could introduce uncertainty in the matching of two fibres. This seems reasonable since the forensic expert could make a mistake when comparing two fibres. One may also think of dealing with the possibility that more than one type of fibre is donated at the transfer moment, for example, jumper fibres and jeans fibres. Finally, in this example we had only once piece of evidence, namely the match of some fibre. In the case that there is more evidence, e.g., blood stains, or footprints, these other pieces of evidence should also be incorporated in the model.

4 The video recognition model

In this section, we construct a model for the following situation. A crime has been committed and thanks to camera surveillance some video material of the criminal is available. This material is then shown to the general public via a television show, like *Opsporing verzocht*.

Obviously, if a person has more acquaintances, the probability that this person is reported is larger. A question that arises naturally is the following. Given a person has been reported, does the probability that he is guilty depend on his number of acquaintances? In other words, should the forensic expert or the judge take into account that the suspect was a very social person, or a very solitary one? In order to answer this, we consider a simple model with the following assumptions:

- The criminal is known to be Dutch and the video material is only shown on Dutch television.
- There is a group of l look-alikes in the Netherlands. These are people who cannot be distinguished from the person on the video by any means.
- One of the look-alikes, called ξ, has n acquaintances.
- Each of these acquaintances reports ξ, independently, with a probability p.

Define the following events:

$$\begin{aligned} S &:= \text{“}\xi \text{ is the person on the video”}, \\ R &:= \text{“}\xi \text{ is reported”}. \end{aligned}$$

Applying Bayes' rule, we find

$$P(S \mid R) = P(S)\frac{P(R \mid S)}{P(R)} = P(S) = \frac{1}{l}, \tag{3}$$

where the second equality holds since all look-alikes look like the person on the video, no matter whether they really are him/her or not. The last identity holds true since there are l look-alikes in the Netherlands.

We see that in this model there is no dependence on either n or p, because we treat all look-alikes as indistinguishable. If we let go of this condition, we get a slightly more sophisticated model, with the following new assumptions.

- The criminal is known to be Dutch and the video material is only shown on Dutch television.
- Some person called ξ has n acquaintances.
- $P(\text{“one acquaintance reports } \xi\text{”} \mid S) = p$.
- $P(\text{“one acquaintance reports } \xi\text{”} \mid S^c) = q$.

Typically, we have $q \leq p$. Under these new assumptions, applying Bayes' rule (2) gives

$$P(S \mid R) = \Big(1 + \frac{P(S^c)}{P(S)} \frac{P(R \mid S^c)}{P(R \mid S)}\Big)^{-1}. \tag{4}$$

From

$$P(R \mid S) = 1 - (1 - P(\text{"one acquaintance reports } \xi\text{"} \mid S))^n = 1 - (1-p)^n,$$
$$P(R \mid S^c) = 1 - (1 - P(\text{"one acquaintance reports } \xi\text{"} \mid S^c))^n = 1 - (1-q)^n,$$

it follows that

$$P(S \mid R) = \Big(1 + \frac{P(S^c)}{P(S)} \frac{1-(1-q)^n}{1-(1-p)^n}\Big)^{-1}.$$

If $p, q \ll 1/n$, then using Taylor's formula yields

$$P(S \mid R) \approx \Big(1 + \frac{P(S^c)}{P(S)} \frac{q}{p}\Big)^{-1}.$$

Note that n does not play a role in this approximation. On the other hand, if $q \ll 1/n$, $q \ll p$, and $p \gg 1/n$, then approximating $(1-p)^n \approx 0$ yields

$$P(S \mid R) \approx \Big(1 + \frac{P(S^c)}{P(S)} qn\Big)^{-1}.$$

Assuming that $P(S^c) \gg P(S)$, the RHS decreases like $1/n$. So in this case, given that a person is reported, the more acquaintances he has, the less likely it is that he is indeed the person on the video. This outcome seems to be counter-intuitive, but can be explained as follows. The more acquaintances a person has, the more likely it is that he is reported. However, the probability that he is the person on the video, and is reported, namely $P(S)(1-(1-p)^n)$, does not change so much, as we assumed that $(1-p)^n \approx 0$. Hence, the probability that he really is the person on the video given that he is reported, decreases.

5 The influence of negative evidence

In criminal investigations not only positive matching results are found. Usually, negative results are not used in court. The question arises whether this is correct. For instance, in the basic jumper model considered in Section 3, what conclusion can be drawn if we do not find a match? Does this imply that the suspect is not very likely to be the offender, or does it hardly have any influence on the probability that the suspect is guilty?

Observe that the basic jumper model concentrates only on the probability that the suspect is the *donor* of the fibre found on the victim. In this section we extend the basic jumper model in order to concentrate on the probability that the suspect is the *offender*. In many crime scenes there are traces that *could* have been donated by the offender, like fibres. On the other hand, there may also be traces that are *very likely* to have been donated by the offender, like bullets or blood. Further, it is not only the nature of the trace that is important here, but also the exact place where it was found. Fibres found on the neck of a strangled person are more likely to be an offender's trace than fibres found under the shoes.

Let us assume that based on this kind of information, for every trace T the forensic expert can assign a probability h_T that this trace was donated by the offender. In the extended model we define the following possible events:

$$\begin{aligned}
TY &:= \text{“Trace found on the victim is of material } Y\text{”}\\
TO &:= \text{“Trace found on the victim is donated by the offender”}\\
SO &:= \text{“Suspect is the offender”}\\
SY &:= \text{“Suspect can be linked to material } Y\text{.”}
\end{aligned}$$

The assumptions of the basic jumper model still apply to this extended model. However, there is no need to restrict the model to fibre matching. For instance, if we are considering a case of DNA-matching, all we have to do is assume that everybody has only one jumper (DNA-pattern). Again, f_Y and g_Y are the relative frequencies of the suspect respectively the whole population carrying material of type Y at the moment of the trace transfer. In the case of DNA we have $f_Y = 1$, since people are not able to change their DNA profile. For the probability that the suspect is guilty given the evidence of a positive match, we write, using (2):

$$\begin{aligned}
P(SO \mid SY \cap TY) &= \left(1 + \frac{P(SO^c)}{P(SO)}\frac{P(SY \cap TY \mid SO^c)}{P(SY \cap TY \mid SO)}\right)^{-1}\\
&= \left(1 + \frac{P(SO^c)}{P(SO)}\frac{P(SY \mid SO^c)}{P(SY \mid SO)}\frac{P(TY \mid SO^c \cap SY)}{P(TY \mid SO \cap SY)}\right)^{-1}.
\end{aligned}$$

Since we assumed that there is no dependence between someone's blood type (or type of fibre that (s)he is wearing) and his or her criminal intent, we have $P(SY \mid SO^c)/P(SY \mid SO) = 1$. Further,

$$\begin{aligned}
P(TY \mid SO^c \cap SY) &= P(TY \cap TO \mid SO^c \cap SY)\\
&\qquad + P(TY \cap TO^c \mid SO^c \cap SY)\\
&= P(TO \mid SO^c \cap SY)P(TY \mid TO \cap SO^c \cap SY)\\
&\qquad + P(TO^c \mid SO^c \cap SY)P(TY \mid TO^c \cap SO^c \cap SY)\\
&= h_T g'_Y + (1 - h_T) g_Y,
\end{aligned}$$

where g'_Y is the relative occurrence of type Y in the whole population minus our suspect. If the trace consist of extremely rare material (like large DNA samples), then g'_Y may differ substantially from g_Y. In fact, for unique material belonging to the suspect, $g'_Y = 0$.

$$\begin{aligned}
P(TY \mid SO \cap SY) &= P(TY \cap TO \mid SO \cap SY)\\
&\qquad + P(TY \cap TO^c \mid SO \cap SY)\\
&= P(TO \mid SO \cap SY)P(TY \mid TO \cap SO \cap SY)\\
&\qquad + P(TO^c \mid SO \cap SY)P(TY \mid TO^c \cap SO \cap SY)\\
&= h_T f_Y + (1 - h_T) g_Y,
\end{aligned}$$

then gives

$$P(SO \mid SY \cap TY) = \left(1 + \frac{P(SO^c)}{P(SO)} \frac{h_T g'_Y + (1 - h_T) g_Y}{h_T f_Y + (1 - h_T) g_Y}\right)^{-1}. \tag{5}$$

This result demonstrates the importance of h_T. If $h_T = 1$, we are sure that the donor of the trace is the offender. Therefore, if we substitute $h_T = 1$ in (5), we find the equivalent result for the basic jumper model discussed in Section 3. On the other hand, if $h_T = 0$, the likelihood ratio is 1, and the trace gives no information about the offender.

We turn to the case that the trace found on the victim cannot be matched to the suspect. As in the positive case, we find

$$P(SO \mid SY^c \cap TY) = \left(1 + \frac{P(SO^c)}{P(SO)} \frac{P(TY \mid SO^c \cap SY^c)}{P(TY \mid SO \cap SY^c)}\right)^{-1}. \tag{6}$$

Observe that $P(TY \mid SO^c \cap SY^c) = g'_Y$. For the denominator we find

$$\begin{aligned} P(TY \mid SO \cap SY^c) &= P(TO \mid SO \cap SY^c) P(TY \mid TO \cap SO \cap SY^c) \\ &\quad + P(TO^c \mid SO \cap SY^c) P(TY \mid TO^c \cap SO \cap SY^c) \\ &= (1 - h_T) g'_Y. \end{aligned}$$

Here we used that $P(TY \mid TO \cap SO \cap SY^c) = 0$. Combining this with (6) yields

$$P(SO \mid SY^c \cap TY) = \left(1 + \frac{P(SO^c)}{P(SO)} \frac{1}{(1 - h_T)}\right)^{-1}.$$

The likelihood ratio $(1 - h_T) \leq 1$ depends only on h_T. We conclude that in general a suspect is less likely to be an offender if no trace can be matched to the suspect. If the police are quite sure that the trace originates from the offender, the likelihood ratio turns out to be very much in favour of the suspect. However, we have to bear in mind that in this model we assumed that no evidence has been destroyed by the suspect. For instance, if we know that the suspect has destroyed the clothes he was wearing during the crime, a negative result on fibre matching does not have any consequence for the likelihood ratio. This kind of complication is hard to incorporate in the model, since the probability that a suspect destroys his clothes depends highly on his innocence, and is hard to estimate. Luckily, this complication does not occur in case of blood traces, or any other traces that originate from the human body. Since these traces are used in many criminal investigations, the above model may still be useful.

Concluding this section, we remark that in many crime cases a lot of traces are collected. If only one of these traces can be linked to the suspect, this trace will be presented in court as evidence, and the other traces will be left out. This selection of evidence seems to be unfair, since the evidential value of the matching trace can be heavily weakened by all traces that cannot be linked to the suspect, especially if some of them were estimated in advance to have a high probability of being offender's traces. We conclude that selection effects in forensic science can be quite important, and from a statistical point of view, improvements can be made to court room practice.

6 Conclusions

To analyse the effect of the way in which evidence is selected, we have set up a very simple model for the matching of fibres found on the victim and clothes belonging to the suspect. In this model we have shown that many things should be reported in order to properly interpret the evidence. We have shown that, for example, the number of jumpers in the wardrobe and the number of fibres at the crime scene influence the probability that the suspect is the offender. Furthermore, in an extension of this jumper model, we have shown that the evidential value of a match can be heavily weakened by all traces that cannot be linked to the suspect.

We stress that we have proved these results in our probabilistic models, which are far from real life situations. Nevertheless, the results we obtained may guide our reasoning in this matter. We conclude that selection effects in forensic science play an important role, and that efforts should be made to improve the statistical interpretation of evidence in court room practice.

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